IMPRECISION AND UNCERTAINTY IN MANAGEMENT
The Possibilities of Fuzzy Sets and Soft Computing

Honoring the 80th Birthday of Prof C.V. Negoiță

Christer CARLSSON[1]
Christer.Carlsson@abo.fi

ABSTRACT:
Soft Computing builds on fuzzy sets theory, fuzzy logic, optimisation, neural nets, evolutionary algorithms, macro heuristics and approximate reasoning. Soft Computing is focused on the design of intelligent systems to process uncertain, imprecise and incomplete information. Soft Computing methods applied to real-world problems offer more robust, tractable and less costly solutions than those obtained by more conventional mathematical techniques. Classical management science is making the transition to *analytics*, which has the same agenda to support managerial planning, problem solving and decision making in industrial and business contexts but is combining the classical models and algorithms with modern, advanced technology for handling data, information and knowledge. The confluence of these trends offers better ways to deal with imprecision and uncertainty in management.

KEYWORDS: Soft computing, analytics, fuzzy real options.

1. Introduction – Management Science and Fuzzy Sets Theory

This paper has a history and a reason that bridges the past, the present and the future. The history is a paper I wrote called *On the Relevance of Fuzzy Sets in Management Science Methodology* in 1984. This was a time when we tried to make the case for fuzzy sets in management science and as a support theory for managers who plan the future, and solve problems and make decisions in their daily activities.

If we continue the history a bit, a first version of the paper had been presented and discussed at the 11th meeting of the EURO Working Group on Fuzzy Sets at the European Institute for Advanced Studies in Management in Brussels on February 19-20, 1981. The EIASM is the centre for serious research on management in Europe and getting an invitation to run a workshop on fuzzy sets took some negotiation; I was chairing the EURO WG in that period and had to do the negotiating.

[1] Prof. Dr., Institute for Advanced Management Systems Research, Abo Akademi University, 20520 Turku, Finland.

The actual historical starting point was, however, about 10 years earlier. There was an exchange agreement in place between Finland and Romania in 1974 to accept researchers for 2 week visits to get to know useful people for scientific cooperation in both countries, and I was offered to go as an exchange researcher to Bucharest. At that time I was listed by Academy of Finland as a cybernetician in their exchange programs (in fact, my doctoral thesis was in Operational Research, Systems Theory and Management Science but this was classified as cybernetics in the exchange programs).

In Bucharest my first morning was spent at a very high level meeting (on the level of the Academy of Science) and it turned out that the political people could not communicate with me as nobody was fluent in English and I did not know any French. Thus the man at the end of the table gave orders to find somebody who knew English and after a while Dr C.V. Negoiță appeared and told me that the high level people did not know what to do with me and that we should move over to the Academy of Economic Studies and find some nice optimization work to do. At his laboratory there was a bohemian guy who turned out to be a very talented cello player and an extreme mathematician, Dan Ralescu. His first words were – "good, you got saved from the politicians (I got the impression that his meaning was "the people who do not know anything") – do you know anything about fuzzy sets?" Thus we spent two weeks working through the theory and some applications of fuzzy sets; my contribution was to share insights about real world, industrial planning, problem solving and decision making; we also spent time discussing and arguing about the few benefits and multiple shortcomings of the socialist economic system.

The next year – 1975 – there was an international conference on general systems theory in Bucharest, Dr Negoiță was on the organizing committee and made sure that I participated in the conference. In Bucharest I met Hannu Nurmi, who had defended his thesis in Political Science in 1974. One of the results of this conference was that we decided in the fall 1975 to start a study group on fuzzy sets in Finland. The driving force was a mathematician, Professor Olavi Hellman at University of Turku, he was professor of applied mathematics and a keen developer of operational research applications for industry and the Finnish defence forces. He had heard a few things about the theory of fuzzy sets and thought that the mathematics was sloppily done and could be improved. Thus we started work on this by first translating and reading up on the text book by Negoiță and Ralescu ["*Mulțimi vagi și aplicațiile lor*" (1974)] – a bit of a challenge as the book was in Romanian but as Olavi Hellman noted, "mathematics is a universal language". We had a bit of luck, there was a German language lecturer at our university who had studied Romanian as the minor for her Master's degree in Linguistics and she translated the Romanian text to German as an exercise; this we could read and in this way we got a first comprehensive text on the theory of fuzzy sets. One of our first steps was to decide on a Finnish concept for the theory of fuzzy sets and we decided on "sumean joukon teoria" (there were variations over the years, the version "sumea joukko teoria" was interpreted as the theory being fuzzy ("sumea") and was not even proper Finnish). Thus, as a matter of historical origin, the development of fuzzy sets in Finland was inspired by C. V. Negoiță and got initiated through his first book with Dan Ralescu on fuzzy sets.

Management science methodology – and especially operations research that applied the same methodology for engineering problems and theory development – had already in 1981 been under attack for more than a decade for failing to deal with the real world problems managers have to tackle, for oversimplifying decision problems and for spending too much time with mathematically interesting but practically irrelevant solutions to problems that had been simplified to be tractable with management science theory and methodology. The message was simply that management science methodology produced theory and methods that were irrelevant for handling actual management problems. The paper in 1984 argued that fuzzy sets when properly worked into management science methodology would make the models, the algorithms and the theory more relevant and better suited to deal with management problems in practice. The early book by Negoiță and Ralescu[2] addressed the vagueness in the real world – equating that with the quality of being fuzzy – but quickly focused on the context of systems analysis which allowed the authors to work out their storyline in clear and elegant mathematics. Chapter 6 on *Deciding in a Fuzzy Environment* is actually a very early attempt at working out a conceptual framework for decision making when handling imprecise data and information; it is interesting to see that the formulations make much sense when reworked in the modern concepts introduced by Analytics. Negoiță returned to the same issues in a new book *Management Applications of System Theory* in 1979,

[2] C. V. Negoiță and Dan Ralescu, *Applications of Fuzzy Sets to Systems Analysis*, Interdisciplinary Systems Research 11, Birkhäuser Verlag, Basel, 1975.

which was five years earlier than the TIMS Studies book edited by Hans-Jürgen Zimmermann, Lotfi Zadeh and Brian Gaines.

Now, more than thirty years later, we have to admit that we were not successful in bringing it about, that fuzzy sets remained a marginal development in management science and that we have been able to get fuzzy sets based methods accepted only for limited applications, such as multiple criteria optimisation, real options valuation, logistics optimisation, etc. for which there have been algorithmic benefits of allowing the use of fuzzy numbers.

Management science and operations research have also changed over the decades; two major organisations in the field – TIMS and ORSA – merged and became INFORMS to combine the applications oriented research (TIMS) with the algorithms and theory oriented research (ORSA); now the annual INFORMS conferences collect 2÷3.000 participants; in Europe the EURO Association is a sister organisation to INFORMS and the annual EURO conferences also collect 2÷3.000 participants. Both organisations run major, well-established journals with high impact factors (*Management Science* and *European Journal of Operations Research*, respectively) and there are dozens of journals publishing material produced under guidance of management science methodology. The field is alive and well and promotes lively research that activates thousands of researchers. The context is there, then what is needed for fuzzy sets to be relevant (again) for the research that is carried out? Operations Research and Management Science are now in the process of being transformed by (*Business*) *Analytics* which is getting the attention of major corporations and senior management. On our part, we have now for a number of years been promoting *Soft Computing* to the same audience instead of trying to explain fuzzy sets theory and fuzzy logic in the way it was originally done[3].

My storyline tries to show that we should use the Analytics movement to make the case for Soft Computing as a viable base to support strategic decisions. The main argument is that Soft Computing (i) offers a combination of sufficient precision and relevance to resolve the main problems with Management Science for practical decision support, but (ii) builds on a core of mathematical theory building that makes it attractive for research and for Analytics. The next section will give a brief outline of Analytics and Soft Computing, section 3 introduces strategic decisions,

[3] *Op. cit*.

section 4 shows how some real options methods will fit the requirements for decision support, section 5 gives a case illustration of how the methods work, and section 6 offers a summary and conclusions.

2. Analytics and Soft Computing

Analytics is gaining support as an important business function that adds value to management; this movement, that promotes data-driven and analytical decision making, is rather recent. Analytics builds on recent software improvements in information systems that has made data, information and knowledge available in real time in ways that were not possible for managers only a few years ago (Davenport and Harris, 2007). Now INFORMS has found out that the new movement represents both "potential opportunities" and "challenges" to management science and operations research professionals[4]. The methods and the application cases worked out in the Davenport-Harris book are very close to traditional text books on management science methodology, actually so close that a manager probably fails to see any differences, which is why INFORMS finds "challenges".

Soft Computing (introduced by Lotfi Zadeh in 1991) builds on fuzzy sets theory, fuzzy logic, optimisation, neural nets, evolutionary algorithms, macro heuristics and approximate reasoning. Soft Computing is a new and innovative area of research which is focused on the design of intelligent systems to process uncertain, imprecise and incomplete information. Soft Computing methods applied to real-world problems offer more robust, tractable and less costly solutions than those obtained by more conventional mathematical techniques.

Liberatore and Luo[5] state that four factors drive the analytics movement: (i) availability of data, (ii) improved analytical software, (iii) the adoption of a process orientation by organisations, and (iv) managers and executives who are skilled users of information and communication technology. Compared to the early 1980's the last factor is probably the most important driver – there is a new generation of managers and executives in charge of the corporations that are using information technology as part of their daily routines. They work with data, information

[4] M. Liberatore, and W. Luo, "INFORMS and the Analytics Movement: The View of the Membership", *Interfaces*, Vol. 41, No. 6, November-December 2011, pp. 578-589.

[5] *Op. cit*.

and knowledge on a real time basis and they continuously hunt for better and better analytical tools to help give them competitive advantages. They do not necessarily recognize the analytical tools as classical management science algorithms because analytical software (cf. (ii)) has become user-friendly through graphical user interfaces and visualisation of results, which allows them to use analytical methods without knowing too much of the mathematical background. Information technology has made data available on a real time basis – in classical management science work off line and sufficient time always had to be allocated for collecting and processing data for the models and algorithms – which allows online planning, problem solving and decision making. Maybe "allow" is not the right verb as online management work in real time now is more of a necessity to keep up with the competition. The same driver also explains the adoption of a process orientation (cf. (iii)) as management work typically is group and teamwork online and in real time.

Davenport and Harris[6] describe *analytics* as "the extensive use of data, statistical and quantitative analysis, explanatory and predictive models and fact-based management to drive decisions and actions". Liberatore and Luo (2011) interpret this definition as representing three levels of modelling – descriptive, predictive and prescriptive – and stated that management science and operations research typically would focus on advanced analytics, i.e. prescriptive modelling. They also point out that *analytics* would focus on the managerial planning, problem solving and decision process, i.e. the transformation of data into actions through analysis and insight, which in their discussion contributes to the application cases of management science.

The modern movement of analytics appears to offer interesting possibilities and opportunities for soft computing; the movement is data-driven which will require tools for handling imprecision; the movement is focused on managers who need to deal with real world problems, for which available data, information and knowledge are incomplete, imprecise and uncertain and should allow for fast, often intuitive conclusions; the movement builds on improved analytical software that can easily incorporate various tools using fuzzy sets (fuzzy numbers, fuzzy optimisation algorithms, linguistic modelling, etc.).

[6] Thomas H. Davenport and Jeanne G. Harris, *Competing on Analytics*. *The New Science of Winning,* Harvard Business School Press, Boston 2007.

Let us then go back to the insights of 1984 and find out in what way soft computing could apply for the analytics of the 2016.

The classical management science approach is to aim for either a logico-deductive or an inductive system and select the methods accordingly; the logico-deductive system is favoured in the sciences and has been favoured also for management science methodology. One of the pioneers, Russell L. Ackoff[7], outlined the system as a problem-solving methodology that handles research problems (here summarized):

(i) Formulate the problem; listing alternative activities that could be carried out, expected outcomes and formulating a set of criteria for comparing the outcomes;

(ii) Construct or select a model; describing the problem formulation in (i) with the help of a set of formal and stringent concepts;

(iii) Select a system of measurement; quantifying the concepts introduced in (ii) through some appropriate system of measurement and delimiting activity and solution spaces;

(iv) Test the model; checking the technical performance of the quantitative model in (iii) and carrying out a preliminary validation;

(v) Derive a solution from the model; deriving numerical values for the elements of the model in (iii); this constitutes a definite choice of a set of activities, which could be the "best" one possible;

(vi) Validate the model and the solution; testing and controlling both the model and the solution in order to make certain that the model is a formally valid and reliable representation of the problem and that the solution is formally correct;

(vii) Carry out experiments with the model, implement and control the solution; testing the applicability and relevance of both the model and the solution to the problem; continue until the model is either accepted or rejected, or modified and developed in order to better correspond to the formulation and the needs of the real world problem.

Each step in this methodology should form a deductive system: (a) a set of undefined and defined concepts to form the framework, which is developed and specified by (b) a set of assumptions, from which is deduced

[7] Russell L. Ackoff, *Scientific Method. Optimizing Applied Research Decisions*, J. Wiley & Sons, New York 1968.

(c) a set of more or less formal theorems, which are confronted with (d) sets of more or less explicit facts[8].

Throughout the history of management science it has been accepted that the methodology described is – in principle – the correct and best way to find solutions to managerial problems. Management science has been much used to explain great breakthroughs in industry and important innovations in business; it has also been useful for explaining and proving that everything necessary and relevant had been done when unexpected events have caused disasters. Many of these explanations have been given *after* the processes have been carried out, not online and in real time when they would have been most needed and useful. There were several reasons for this: data was not available, there was no time to build and use the necessary decision models and the methodology is the correct and best one, but too time-consuming. Thus management science methodology is – for most practical purposes - not relevant.

This is where we made a case for fuzzy sets in the 1984 paper; the theory of fuzzy sets is developed for a domain in which descriptions of activities and observations are *imprecise*, in the sense that there are no well-defined boundaries of the set of activities or observations to which the descriptions apply.

In management we have learned over the years that *imprecision* differs from "generality", which is the application of one description to a (well-defined) set of activities or observations; it differs also from "ambiguity", which refers to the use of several, competing (but well-defined) descriptions of a set of activities or observations; *imprecision* is not "uncertainty" in the sense of subjective probability theory – because it does not use its axioms – nor in the sense of classical probability theory as it does not build on the frequencies of events, activities, observations, etc.

Why is it important to make this point? It is a well-known fact that it is virtually impossible to give an exact description of any real physical situation; needless to say this holds also for any managerial problem situation, especially as many of these problems relate to an unknown future. A science-oriented methodology knows only exact, well-defined activities, outcomes, external activities and goals – and functional relationships. This is fundamental for the limitations of management science methodology: it has not been possible to give an adequate representation of *imprecision* – or

[8] *Ibidem.*

as Lotfi Zadeh[9] wrote "..., *we need a radically different kind of mathematics, the mathematics of fuzzy or cloudy quantities which are not describable in terms of probability distributions*".

The theory of fuzzy sets allows us to structure and describe activities and observations which differ from each other imprecisely, to formulate them in models and to use these models for managerial problem solving and decision making. We are trained as scientists to use precise concepts and sharp definitions in order to be able to build precise and elegant models, to use mathematically well-defined algorithms so that we can give distinct descriptions, precise explanations and concise predictions; as managers we are trained to use precise concepts and sharp definitions so that we can give distinct descriptions of business events, precise explanations of business opportunities and concise predictions of the outcomes when using resources to achieve business ends; or at least we strive towards these ideals. The experience we now have (which is also supported by analytics) is that this ability is not the most important one for handling managerial problems.

We humans are said to think and reason in imprecise, non-quantitative, vague terms which gives us the ability (i) to summarize, (ii) to focus on relevant information and knowledge and (iii) to concentrate on the essence when handling very large amounts of data and information (a by-product of the information technology revolution). These three capabilities are often cited as "essential" for managers-to-be.

Analytics has a similar agenda as management science and is working with the same industrial and business context to support managerial planning, problem solving and decision making. Analytics has a broader scope in terms of methods – besides models and algorithms it also works with statistical methods and advanced technology for handling data, information and knowledge. The software used for analytics is several generations more advanced than the software used for management science in 1984. The manager for whom analytics support is developed is a generation more advanced in using information technology and modelling tools. Data, information and knowledge are available in real time for online use to support fast moving business operations. Davenport and Harris[10]

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[9] Lotfi A. Zadeh, "Fuzzy Sets", *Information and Control*, 8 (3), 1965, pp. 338-353.

[10] Thomas H. Davenport and Jeanne G. Harris, *Competing on Analytics*. *The New Science of Winning, cited edition.*

argue that "*sophisticated quantitative and statistical analysis and predictive modelling supported by data-savvy senior leaders and powerful information technology*" are the key elements of competitive strategies that make the difference between winning and losing business.

3. Soft Computing and Strategic Decisions

We will now work through some arguments why Soft Computing is useful as a support for management that is forced to make strategic or hard decisions.

Hard decisions will have significant economic, financial, political and/or emotional consequences for a management team and the company they serve. Hard decisions are normally difficult to make and are made even harder if the decision situation is complex (i.e. there are many interdependent elements), the information about the decision alternatives and their consequences is imprecise and/or uncertain and the environment (or the context) unstable, dynamic and not well known. If a team or a group should make the decisions the group members may have different opinions about the alternatives, the risks or the outcomes.

We will support our argument with data and experience from a real world case – the strategic decision on the closing/not closing of a paper mill in the UK owned by a Finnish forest corporation. We worked with the management team during an 18 month period and followed the processes they went through and tried to support them with good analytical tools as best we could. In this way we gained a fairly good understanding of how management works with hard decisions.

The decision is made hard by several opposing and competing views: the responsibility to the shareholders is a good argument for closing the plant, the responsibility to the employees and the community where the plant has been operating for nearly a century is a good argument for not closing the plant. Then we have the overall market situation and the profitability development for the European forest industry, the different results skilful people get with different analytical tools and the different market trends people believe in. Still the management team needs to find a good (or preferably the best) decision to recommend to the board of directors - a good decision can be explained in logical and analytical terms with a good support of facts and can be explained with rational arguments; the best decision is simply dominating any other alternative that can be discussed or tested. The management team needs a bit more than that – they need to be able to understand all the alternatives and their consequences, they need to be able to analyse and understand the alternatives with all the data that is available, they need to have a reasonable foresight into the coming markets, they need to be able to discuss the issues and the alternatives in terms they can understand jointly and they need to come to a consensus on what they should be doing.

The paper mill has had an unsatisfactory profitability development for a number of reasons: (i) fine paper prices have been going down for six years, (ii) costs are going up (raw material, energy, chemicals), (iii) demand is either declining or growing slowly depending on the markets, (iv) production capacity cannot be used optimally, and (v) the £/USD exchange rate is unfavourable (sales invoiced in USD, costs paid in £). The standard solution for most forest industry corporations is to try to close the old, small and least cost-effective production plants. The plant is producing fine paper products, it is rather aged, the paper machines were built a while ago, the raw material is not available close by, energy costs are reasonable but are increasing in the near future, key domestic markets are close by and export markets (with better sales prices) will require investments in the logistics network.

The intuitive conclusion is that we have a sunset case and senior management should make a simple, executive decision and close the plant. On the other hand we have the UK trade unions, which are strong, and we have pension funds commitments until 2013 which are very strict, and we have long-term power contracts which are expensive to get out of. Finally, by closing the plant we will invite competitors to fight us in the UK markets we have served for more than 50 years and which we cannot serve from other plants at any reasonable cost.

It is clear that the decision problem is more complex than the standard formulations we use in management science and that a number of factors that will decide the outcome are not easily handled with algorithms.

There were a number of conditions which were more or less predefined. The *first* one was that no capital could/should be invested as the plant was regarded as a sunset plant. The *second* condition was that we should in fact consider five scenarios: the current production setup with only maintenance of current resources and four options to switch to setups that save costs and have an effect on production capacity used. The *third* condition is that the plant together with another unit is carrying and should carry considerable administrative costs of the sales organization in the country and if the plant is closed these costs have to be covered in some way (but not clear how). The *fourth* condition is the pension scheme that needs to be financed until 2013. The *fifth* condition is given by the power contracts that are running until 2013. These specific conditions have consequences on the cost structure and the risks that various scenarios involve. It is not known if the conditions are truly non-negotiable. The management team should decide if the plant will (i) be closed as soon as possible, (ii) not be closed, or (iii) be closed at some later point of time (and then at what point of time).

Profitability analysis has usually had an important role as the threshold phase and the key process when a decision should be made on closing or not closing a plant. Economic feasibility is a key factor but more issues are at stake. Management decisions will be scrutinized and questioned regardless of what the close/not close decision is going to be. The shareholders will react negatively if they find out that share value will decrease (closing a profitable plant, closing a plant which may turn profitable, or *not* closing a plant which is not profitable, or which may turn unprofitable) and the trade unions, local and regional politicians, the press etc. will always react negatively to a decision to close a plant almost regardless of the reasons.

Modern profitability analysis is usually built with methods that originate in neoclassical finance theory. These models are by nature normative and may support decisions that in the long run may be proved to be optimal but may not be too helpful for real life decisions in a real industry setting as conditions tend not to be well structured as in theory.

In profitability planning a good enough solution is many times both efficient, in the sense of smooth management processes, and effective, in the sense of finding the best way to act, as compared to theoretically optimal outcomes. The case for good enough solutions is made in fuzzy set theory[11]: *at some point there will be a trade-off between precision and relevance, in the sense that increased precision can be gained only through*

[11] Christer Carlsson and Robert Fullér, *Fuzzy Reasoning in Decision Making and Optimization*, Springer Verlag, Berlin-Heidelberg, 2002; C. Carlsson and R. Fullér, "A Fuzzy Approach to Real Option Valuation", *Fuzzy Sets and Systems*, 139, 2003, pp. 297-312; C. Carlsson, Fullér R and Majlender P., "A Fuzzy Real Options Model for R&D Project Evaluation", in: *Proceedings of the 11th IFSA World Congress*, 2005.

loss of relevance and increased relevance only through the loss of precision.

Only very few decisions are of the type *now-or-never* – often it is possible to postpone, modify or split up a complex decision in strategic components, which can generate important learning effects and therefore essentially reduce uncertainty. If we close a plant we lose all alternative development paths which could be possible under changing conditions. These aspects are widely known – they are part of managerial common wisdom – but they are hard to work out unless we have the analytical tools to work them out and unless we have the necessary skills to work with these tools.

We chose to work with real options models as our analytical tools for the paper mill. The rule we worked out, is that *we should only close the plant now if the net present value of this action is high enough to compensate for giving up the value of the option to wait*. Because the value of the option to wait vanishes right after we irreversibly decide to close the plant, this loss in value is actually the opportunity cost of our decision[12]. This is a principle based in theory but it turned out that the principle was well understood by the management team. The mathematics involved in working with real options modelling is fairly advanced but we worked it out with the managers in a series of workshops where we also introduced and demonstrated the software (actually Excel models) we were using – the key turned out to be that we used the management team's own data to explain the models step by step. They could identify the numbers and fit them to their own understanding of the close/no close problem and the possible problem solving paths shown by the real options models.

4. Soft Computing and Fuzzy Real Options

The value of a real option is computed[13]

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[12] Francisco Augusto Alcaraz Garcia, *Real Options, Default Risk and Soft Applications*, TUCS Dissertations, 82, 2006 ; E. Borgonovo and L. Peccati, Sensitivity analysis in investment project evaluation, *International Journal of Production Economics*, 90, 2004, pp. 17-25; Markku Heikkilä, *R&D Investment Decisions with Real Options – Profitability and Decision Support*, Åbo Akademi University Press, Åbo, 2009.

[13] C. Carlsson and R. Fullér, "A Fuzzy Approach to Real Option Valuation", *Fuzzy Sets and Systems*, 139, 2003, pp. 297-312; C. Carlsson, Fullér R and Majlender P., "A Fuzzy Real Options Model for R&D Project Evaluation", in: *Proceedings of the 11th IFSA World Congress*, 2005; M. Collan, C. Carlsson and P. Majlender, "Fuzzy Black and Scholes Real

$$\mathrm{ROV} = S_0 e^{-\delta T} N(d_1) - X e^{-rT} N(d_2),$$

where

$$d_1 = \frac{\ln(S_0/X) + (r - \delta + \sigma^2/2)T}{\sigma\sqrt{T}},$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Here, S_0 denotes the present value of the expected cash flows, X stands for the nominal value of the fixed costs, r is the annualized continuously compounded rate on a safe asset, δ is the value lost over the duration of the option, σ denotes the uncertainty of the expected cash flows, and T is the time to maturity of the option (in years). The interpretation is that we have the difference between two streams of cash flow: the S_0 is the revenue flow from the plant and the X is the cost generated by the plant; both streams are continuously discounted with a chosen period of time T and the streams are assumed to show random variations, which is why we use normal distributions N. In the first stream we are uncertain about how much value we will lose δ if we postpone the decision and in the second stream we have uncertainty on the costs σ.

The function $N(d)$ gives the probability that a random draw from a standard normal distribution will be less than d, i.e. we want to fix the normal distribution,

$$N(d) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{d} e^{-x^2/2}\, dx.$$

Facing a deferrable decision, the main question that a company primarily needs to answer is the following: h*ow long should we postpone the decision - up to T time periods - before (if at all) making it?*

Options Pricing”, *Journal of Decision Systems*, 12, 2003, pp. 391-416; Mikael Collan, *Giga-Investments: Modeling the Valuation of Very Large Industrial Real Investments*, TUCS Dissertations, Turku, 2004.

With the model for real option valuation we can find an answer and develop the following natural decision rule for an optimal decision strategy; again this requires a bit of analytical modelling[14].

Let us assume that we have a deferrable decision opportunity P of length L years with expected cash flows $\{cf_0, cf_1, \ldots, cf_L\}$, where cf_i is the cash inflows that the plant is expected to generate at year i ($i = 0, \ldots, L$). We note that cf_i is the anticipated net income (revenue – costs) of decision P at year i. In these circumstances, if the maximum deferral time is T, we shall make the decision to postpone for t' periods (which is to exercise the option at time t', $0 < t' < T$) for which the value of the option, $ROV_{t'}$ is positive and gets its maximum value; namely[15],

$$ROV_{t'} = \max_{t=0,1,\ldots,T} ROV_t = \max_{t=0,1,\ldots,T} V_t e^{-\delta T} N(d_1) - Xe^{-rT} N(d_2) > 0,$$

If we make the decision now without waiting, then we will have:

$$ROV_0 = V_0 - X = \sum_{i=0}^{L} \frac{cf_i}{(1+\beta_P)^i} - X.$$

That is, this decision rule also incorporates the net present valuation of the assumed cash flows; β_P is the risk-adjusted discount rate of the decision. This is the rule for how long we can postpone the decision to close/not close the production plant which is anchored in solid economic theory (thus we can give a rational motivation for the decision). The real option model actually gives a value for the deferral which makes it possible to find the optimal deferral time. The management team now has an instrument for the hard decision.

Having got this far we will have to face another problem: the difference between management science modelling and what is possible with the real world case. Real options theory requires rather rich data with a good level of precision on the expected future cash flows. This is possible for financial options and the stock market as we have the effective market hypothesis which allows the use of models that apply stochastic processes and which have well known mathematical properties. The data we could

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[14] Carlsson C and Fullér R (1999), Capital budgeting problems with fuzzy cash flows, *Mathware and Soft Computing*, 6, 81-89; C. Carlsson and R. Fullér, "A Fuzzy Approach to Real Option Valuation", *Fuzzy Sets and Systems*, 139, 2003, pp. 297-312 .

[15] For details, see C. Carlsson and R. Fullér, "A Fuzzy Approach to Real Option Valuation", *Fuzzy Sets and Systems*, 139, 2003, pp. 297-312.

collect on the expected future cash flows of the paper mill were not precise and were incomplete; the management team was rather reluctant to offer any firm estimates (for very understandable reasons, these estimates can be severely questioned with the benefit of hindsight). It turns out that we could work out the real options valuation also with imprecise and incomplete data.

Let us now assume that the expected cash flows of the close/not close decision cannot be characterized with single numbers. With the help of possibility theory[16] we can estimate the expected incoming cash flows at each year of the project by using a trapezoidal possibility distribution of the form

$$\overline{V_i} = (s_i^L, s_i^R, \alpha_i, \beta_i), \quad i = 0,1,\ldots,L,$$

that is, the most possible values of the expected incoming cash flows lie in the interval $[s_i^L, s_i^R]$ (which is the core of the trapezoidal fuzzy number describing the cash flows at year i of the paper mill); $(s_i^R + \beta_t)$ is the upward potential and $(s_i^L - \alpha_t)$ is the downward potential for the expected cash flows at year i, (i = 0, 1, …, L). In a similar manner we can estimate the expected costs by using a trapezoidal possibility distribution of the form

$$\overline{X} = (x^L, x^R, \alpha', \beta'),$$

i.e. the most possible values of the costs lie in the interval $[x^L, x^R]$; $(x^R + \beta')$ is the upward potential and $(x^L - á')$ is the downward potential for the expected fixed costs (this is of course a simplification, there should be different costs for each year, but the management team stated that they do not change much and that the trouble of estimating them does not have a good trade-off with the accuracy of the model).

By using possibility distributions we can extend the classical probabilistic decision rules for an optimal decision strategy to a possibilistic context.

We will now revisit our decision rule when the model is built with fuzzy numbers. Let P be a deferrable decision opportunity with incoming cash flows and costs that are characterized by the trapezoidal possibility distributions given above. Furthermore, let us assume that the maximum deferral time of the decision is T, and the required rate of return on this project is β_P. In these circumstances, we should make the decision (exercise

[16] *Ibidem.*

the real option) at time *t*', 0 < *t*'< *T*, for which the value of the option, $C_{t'}$ is positive and reaches its maximum value. That is,

$$\overline{FROV_{t'}} = \max_{t=0,1,\ldots,T} \overline{FROV_t} = \max_{t=0,1,\ldots,T} \overline{V_t} e^{-\delta t} N(d_1^{(t)}) - \overline{X} e^{-rt} N(d_2^{(t)}) > 0,$$

where:

$$d_1^{(t)} = \frac{\ln\left(E(\overline{V_t}) / E(\overline{X})\right) + \left(r - \delta + \sigma^2/2\right)t}{\sigma\sqrt{t}},$$

$$d_2^{(t)} = d_1^{(t)} - \sigma\sqrt{t} = \frac{\ln\left(E(\overline{V_t}) / E(\overline{X})\right) + \left(r - \delta - \sigma^2/2\right)t}{\sigma\sqrt{t}}.$$

Here, *E* denotes the possibilistic mean value operator and:

$$\sigma = \sigma(\overline{V_t}) / E(\overline{V_t})$$

shows the annualized possibilistic variance of the aggregate expected cash flows relative to its possibilistic mean. Furthermore,

$$\overline{V_t} = \mathrm{PV}(\overline{cf_0}, \overline{cf_1}, \ldots, \overline{cf_L}; \beta_P) - \mathrm{PV}(\overline{cf_0}, \overline{cf_1}, \ldots, \overline{cf_{t-1}}; \beta_P) = \mathrm{PV}(\overline{cf_t}, \ldots, \overline{cf_L}; \beta_P) = \sum_{i=t}^{L} \frac{\overline{cf_i}}{(1+\beta_P)^i}$$

computes the present value of the aggregate (fuzzy) cash flows of the project if this has been postponed *t* years before being undertaken.

To find a maximizing element from the set:

$$\left\{\overline{FROV_0}, \overline{FROV_1}, \ldots, \overline{FROV_T}\right\}$$

we need to have a method for the ordering of trapezoidal fuzzy numbers. This is one of the partially unsolved problems for fuzzy numbers as we do not have any complete models for ranking intervals (cf. Carlsson and Fullér (2003) for details), which is why we have to resort to various ad hoc methods to find a ranking. Basically, we can simply apply some value function to order fuzzy real option values of trapezoidal forms:

$$FROV_t = (c_t^L, c_t^R, \alpha_t', \beta_t'), \quad t = 0,1,\ldots,T.$$

$$v(FROV_t) = \frac{c_t^L + c_t^R}{2} + r_A \cdot \frac{\beta_t' - \alpha_t'}{6},$$

where r_A ∃ 0 denotes the degree of the manager's risk aversion. If $r_A = 1$ then the manager compares trapezoidal fuzzy numbers by comparing their pure possibilistic means (cf. Carlsson and Fullér (2001)). Furthermore, in the case $r_A = 0$, the manager is risk neutral and compares fuzzy real option values by comparing the centre of their cores, i.e. he does not care about their upward or downward potentials.

Thus we can work out the best time for making a close/not close decision on the paper mill also with imprecise and incomplete data.

5. Analytics and Soft Computing in Practice

The paper mill case is an evaluation of the current production setup with four scenarios (Scenario 1-4) for future development; in the work, the management team also wanted to try out variations to Scenario 1 (labelled as 1A and 1B); each scenario worked with Product 1-3, which also represented their own production lines. Some details of the support system built around the real options models are still confidential but the main principles can be shown.

For the actual analysis we used the binomial version of the fuzzy real options model[17] as this was easier to implement as an analysis instrument; the actual tools were Excel models to which we connected the @Risk add-on module for risk modelling and simulation. One of the lessons learned from the case is that there is no added value in using advanced models that may not be fully understood by the management if we want to have well understood decision support with sufficient precision.

Cash flows were estimated for each of the sales scenarios of the three production lines accounting for the changes in the fixed costs caused by the production scenarios. Each of the products had their own price forecast that was used as a trend factor. For the estimation of the cash flow volatility there were two alternative methods of analysis. Starting from the volatility of sales price estimates we can get the volatility of cash flow estimates by simulation (the Monte Carlo method) or by using the management team's opinions as added value estimates.

[17] For details, M. Heikkilä and C. Carlsson, "A Fuzzy Real Options Model for (Not) Closing a Production Plant: An Application to Forest Industry in Finland", *Proceedings of the 12th Annual International Conference on Real Options*, Rio de Janeiro, 2008; Markku Heikkilä, *R&D Investment Decisions with Real Options – Profitability and Decision Support,* Åbo Akademi University Press, Åbo, 2009.

It turned out that the added value estimates (AVE) are more robust for planning purposes than individual revenue and cost estimates that could be allocated to Products 1-3. Calculating the AVE requires access to the actual revenue and cost data of the plant; this data cannot be shown as it is highly confidential. We have modified the AVE with a random factor in order not to reveal the actual state of the plant. The data shown may appear to be rather dated but the models and the analysis had to be kept confidential for five years; the actual numbers are not essential for an understanding of the decision problem.

It turned out that the management team was both rather good at making the estimates and willing to make them as there was an amount of flexibility in using the (trapezoidal) fuzzy numbers (cf. fig. 1).

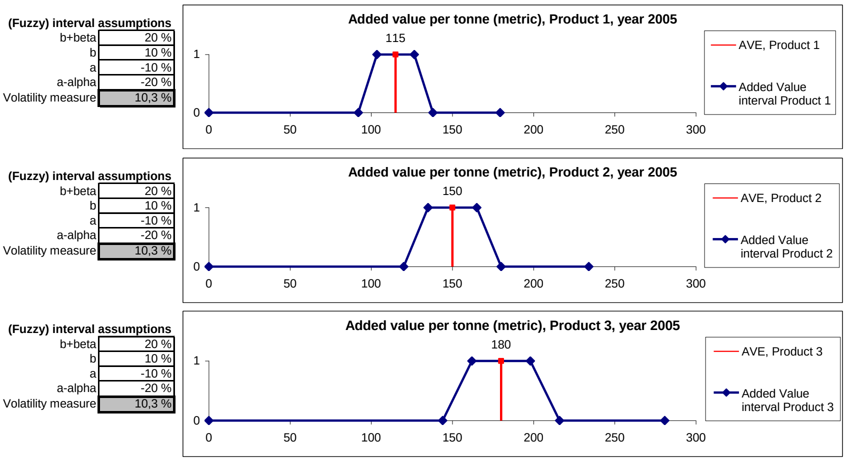

Figure 1. Added value estimates, trapezoidal fuzzy interval estimates and retrieved volatilities

The annual cash flow in the option valuation is the cash flow of postponing the switch of production from which was subtracted the cash flow of switching now. The resulting cash flow of switching immediately is shown below (cf. fig.2). The cash flows were transformed from nominal to risk-adjusted in order to allow risk-neutral valuation (this refinement was asked for by a plant controller who wanted to make a point). The management team could trace and intuitively validate the numbers as "reasonable".

Year	0	1	2	3	4	5
Fixed Cost Total, Scenario 1A	***0***	***-5 620 750***	***-5 757 269***	***-5 899 200***	***-6 056 180***	***-6 257 835***
Added Value Total, Scenario 1A	***0***	***6 465 000***	***7 358 000***	***7 913 000***	***8 881 000***	***8 902 000***
EBDIT, Scenario 1A	***0***	***844 250***	***1 600 731***	***2 013 800***	***2 824 820***	***2 644 165***
Risk-neutral valuation parameter	1,000	0,955	0,911	0,870	0,830	0,792
EBDIT	**0**	**805 875**	**1 458 518**	**1 751 484**	**2 345 185**	**2 095 423**
NPV, no delay	***7 174 624***	**8 148 015**				

Figure 2. Incremental cash flows and NPV with no delay in the switch to Scenario 1A

The switch immediately to Scenario 1A (cf. fig.2) seems to be profitable. In the following option value calculation the binomial process results are applied in the row "*EBDIT, from binomial EBDIT lattice*". The calculation shows that when the given volatilities are applied to the products and the retrieved Added Value, the resulting EBDIT lattice returns cash flow estimates for the *option to switch*, which adds 24 million of managerial flexibility (cf. fig. 3).

Year	0	1	2	3	4	5	6
Fixed Cost Total, Scenario 1A	***0***	***-5 620 750***	***-5 757 269***	***-5 899 200***	***-6 056 180***	***-6 257 835***	***-6 390 171***
Added Value Total, Scenario 1A	***0***	***6 465 000***	***7 358 000***	***7 913 000***	***8 881 000***	***8 902 000***	***8 786 900***
EBDIT, Scenario 1A	***0***	***844 250***	***1 600 731***	***2 013 800***	***2 824 820***	***2 644 165***	***2 396 729***
Risk-neutral valuation parameter	1,000	0,955	0,911	0,870	0,830	0,792	0,756
EBDIT	**0**	**805 875**	**1 458 518**	**1 751 484**	**2 345 185**	**2 095 423**	**1 813 003**
NPV, no delay	***7 174 624***	**8 148 015**					
NPV at year 2006		**7 777 651**					
NPV,delay: 1 year(s)		**603 027**					
EBDIT, from binomial EBDIT lattice		**3 711 963**	**6 718 118**	**8 067 557**	**10 802 222**	**9 651 783**	**12 064 213**
Option to switch, value at year 2006		**33 047 232**					
Option to switch		**31 545 085**					
Flexibility		**24 370 461**					

Figure 3. Incremental cash flows, the NPV and Option Value Assessment when the switch to Scenario 1A is delayed by 1 year

The fuzzy interval analysis allows management to make scenario-based estimates of upward potential and downward risk separately. The volatility of cash flows is defined from a possibility distribution and can readily be manipulated if the potential and risk profiles of the project change. Assuming that the volatilities of the three product-wise AVEs were different from the ones presented in fig. 1 to reflect a higher potential of Product 3 and a lower potential of Product 1, the following volatilities could be retrieved (cf. fig.4Figure). Note that the expected value with products 1 and 3 now differs from the AVEs.

The fuzzy cash flow based profitability assessment allows a better analysis of the sources of a scenario value. In real option analysis such an asymmetric risk/potential assessment is realised by the fuzzy ROV. Added values can now be presented as fuzzy added value intervals instead of single (crisp) numbers. The intervals are then run through the whole cash flow table with fuzzy arithmetic operators. The fuzzy intervals described in this way are trapezoidal fuzzy numbers (cf. fig. 4).

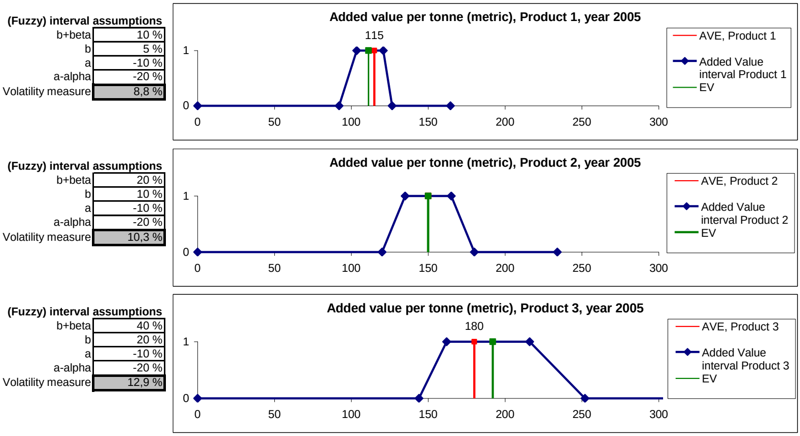

Figure 4. Fuzzy Added Value intervals and volatilities

In the Excel models we decided to calculate the net present value (NPV), which is the standard way of comparing scenarios which are built around assumptions of future cash flows. This proved to be a good way to improve the understanding of how the fuzzy real option valuation (ROV) is built and used. As a result from the analysis a NPV calculation now supplies the results of the NPV and fuzzy ROV as fuzzy numbers. Also flexibility is shown as a fuzzy number (cf. fig.5).

This comparative analysis is made by applying a standard volatility (10.3%) for each product, scenario and option valuation method. Fig. 5 shows that the NPV does not support postponing the decision but the fuzzy ROV recommends a delay of 2 years. We also worked out a simple model to allow the management team to experiment with switching to Scenario 1A at different years (cf. fig. 6).

	2004	2005	2006	2007
Present value at delay		7,174,624		6,494,629
Present value at delay, Support up		9,834,912		14,886,532
Present value at delay, Core up		7,552,125		11,824,291
Present value at delay, Core down		2,986,552		5,699,809
Present value at delay, Support down		703,765		2,637,568
Present value at delay, Fuzzy EV		6,410,732		10,293,171
Present value at delay, St. Dev.		2,345,340		3,146,154
Present value at delay, St. Dev. %		36.6%		30.6%
NPV at present year, 2005	Flexibility			
Delay value without flexibility	-1,283,804	7,174,624		5,890,820
Delay value with flexibility, Support Up	3,667,612	9,834,912		13,502,524
Delay value with flexibility, Core Up	3,172,855	7,552,125		10,724,981
Delay value with flexibility, Core Down	2,183,343	2,986,552		5,169,895
Delay value with flexibility, Support Down	1,688,587	703,765		2,392,352
Delay value with flexibility, Fuzzy EV	2,925,477	6,410,732		9,336,209
Delay value with flexibility, St. Dev.	508,314	2,345,340		2,853,654
Delay value with flexibility, St. Dev. %	17.4%	36.6%		30.6%
Delay	2			

Figure 5. Fuzzy interval assessment, NPV and fuzzy Real Option Value (ROV)

In this way we worked through all the combinations of Products 1-3 and Scenarios 1-4, and even tested some variations like Scenario 1A and 1B. We came to the conclusion that there is a positive option value in delaying the closing of the paper mill at least until the year 2010. This contradicted the results we got with the NPV methods which recommended closing the plant in the next 1-3 years for all scenarios.

Overall it is fair to say, that the analysis shows that there are viable alternatives to an immediate closing of the paper mill, and that there are several options for continuing with the current operations. The uncertainties in the added value processes show significantly different results when, on the one hand, both risk and potential are aggregated to one single number in the binomial process (which is the traditional way) and, on the other hand, there is a fuzzy number that allows the treatment of the downside and the upside differently. The specific costs of a closedown (the pension scheme and the energy contracts) are large opportunity costs for an immediate closedown.

The developed models support screening alternative paths of action as options. We found that the binomial assessment, which is based on assumptions of real asset tradability, overestimates the real option value, and gives the management flexibilities that actually are not there. The fuzzy cash flow interval approach allows an interactive treatment of the uncertainties on the (annual) cash flow level and in that sense gives the management powerful decision support. The fuzzy real options method offers both rigor and relevance as we get a normative profitability analysis with readily available uncertainty and sensitivity assessments.

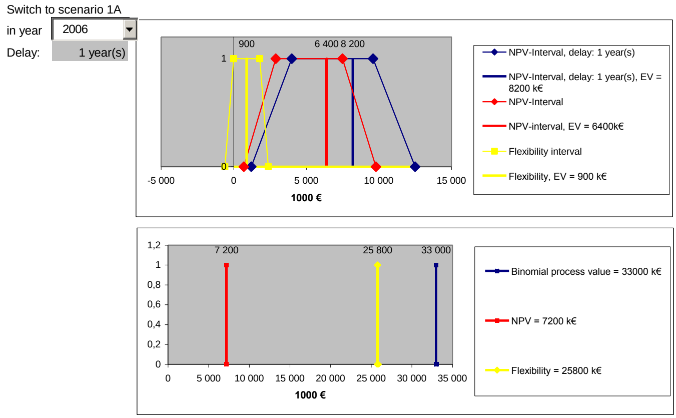

Figure 6. Comparing the results graphically, the option to Switch to Scenario 1A at 2006

The paper mill was closed on January 3, 2007 at significant cost according to our analysis; in 2009 we found out that the senior manager – the head of the management team with which we worked – was able to negotiate a more reasonable deal with the trade unions and the power companies and the actual cost was not as high as our analysis showed (he used our results as a benchmark for the negotiations).

5. Summary and Conclusions

The introduction of fuzzy sets theory (or the sales work, if you like) as a key element of management science research more than thirty years ago did not meet with success. We tried to show that the handling of *imprecision* is not done properly in management science research and that fuzzy sets theory offers a proper and effective theory basis.

As the Analytics movement is gaining support and is readdressing some of the problems with management science we now have an opportunity to show that fuzzy sets would be useful as a theory basis. The way to accomplish that is to make Soft Computing a vital part of Analytics, because it (i) offers a combination of sufficient precision and relevance to resolve the main problems with Management Science, and it (ii) builds on a core of mathematical theory that is attractive for Analytics.

We worked out a storyline built around strategic decisions to demonstrate the combination of Analytics and Soft Computing. We worked through an actual case to show how real world management issues are dealt with in an analytics framework and how soft computing contributes to a resolution of the problems with imprecision. We found both a generic principle for how to deal with the strategic decision and how to build an effective decision support with the help of fuzzy real options models.

Dr. C.V. Negoiță was one of the early pioneers and advocates for the use of fuzzy sets in management. The context for the use of analytical methods in management at that time (the 1970's) was guided by the key assumptions used to build management science and operational research models (also known as cybernetics models) for handling complex and challenging planning, problem solving and decision problems. The context was also defined by the limitations of computational technology – there were a number of options found with the mathematical tools that could not be implemented because they could not be worked out with the hardware and software available in the 1970's. We have now been able to work out the insights and the ideas of the pioneers with the modern tools introduced with soft computing and analytics – and computational technology much more advanced than we could even dream of in the 1970's – and we start to see the real impact we can make on handling imprecision and uncertainty in management in actual operations (even through mobile technology), in short- and mid-term planning and in strategic planning[18]. The legacy of the early work published by C.V. Negoiță and his co-workers lives on and is bearing fruit on a scale that could not have been anticipated.

[18] C. Carlsson, M. Heikkilä and J. Mezei, "Possibilistic Bayes Modelling for Predictive Analytics", *Proceedings of 15th IEEE International Symposium on Computational Intelligence and Informatics*, Budapest, Nov 2014, pp. 15-20; C. Carlsson, J. Mezei and R. Wikström, "Aggregating Linguistic Expert Knowledge in Type-2 Fuzzy Ontologies", *Applied Soft Computing*, Vol 35, October 2015, pp 911-920; C. Carlsson, M. Heikkilä and J. Mezei, "Fuzzy Entropy Used for Predictive Analytics", in Cengiz Kahraman (ed.), *Fuzzy Sets in its 50th Year. New Developments, Directions and Challenges,* Studies in Fuzziness, Vol. 341, Springer, 2016, 23 pp.

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