OPERATING ON SYMMETRY AND SCALE TO ADDRESS ENVIRONMENTAL VARIABILITY

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ABSTRACT:
This essay focuses on questions concerning the windows we open towards the environment, such as our orientation in space and in time, the identification of spatial and temporal features, and the design of methodological tools meant to grasp the behaviour of natural systems. We show that symmetry can be attached to the studied systems both in space and in time, with significant positive effects on our exploration endeavours. The removal of symmetry networks often leaves behind a different, yet important form of symmetry: scale invariance. The latter is shown to represent a valuable property of the natural environment, one that plays an important role in our effort to understand reality.

KEYWORDS: symmetry, scale, scale invariance, space, time, environment.

1. Some reference elements

It is difficult to find concepts that are deeper in the realm of meanings and more pervasive on human horizons than space and time. It would thus seem audacious to attempt an overview – albeit concise – of their role in our investigations regarding the environment. The aim of the paper is simply to follow some implications of addressing spatial and temporal patterns by the addition or removal of symmetry fingerprints, especially in interaction with changes in scale.

Among the two concepts in the title, symmetry and scale, the former is mostly associated with the studied systems themselves, although it may also be part of our investigation tools, as discussed below. In contrast, scale is expected to primarily represent a property of the instrument, the map, the "lens" applied to explore the environment. At the same time, scale is also related to the systems under investigation, at least due to the size of such systems, which makes only certain scale ranges meaningful[2]. Both

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[2] One should distinguish the scales on which systems are described from those on which the systems operate; these scales can be very different from each other: consider, for example, an asteroid and its orbit in space.

concepts, symmetry and scale, will be considered here mainly from the point of view of their epistemological value. In particular, we will focus on processes and implications of turning symmetry and scale into exploration tools, as well as on their multiple forms of interaction: in fact, it is often their interaction that is decisive for the outcome of our epistemic relations to the environment. The importance and usefulness of these concepts are probably among the reasons for the diversity of meanings associated with them in daily life, on one hand, and in various scholarly fields, on the other hand. We are thus compelled to add a clarifying, brief outline of the way in which they will be applied in this essay.

Symmetry represents more than a key feature of the surrounding world, and more than a mere intellectual tool. According to Hermann Weyl, "symmetry […] is one idea by which man through the ages has tried to comprehend and create order, beauty and perfection"[3]. Symmetry plays a fundamental role in science, dominating both its mathematical apparatus and the "real world" to which the mathematical tools are applied: "we view Nature through symmetry spectacles and understand nature in the language of symmetry"[4]. The diversity of views on symmetry is very rich, and identifying the common core of the sparkling auras it enjoys in various fields[5] is not easy to grasp in a spatially constrained context. We will not try to delineate such a core, but rather confine our approach to ways in which it is defined and applied in science. Moreover, our endeavour is not meant to address the role of symmetry in science in general (insightful books have been dedicated to this goal): we will focus on symmetry as an investigation instrument. Symmetry is commonly presented as a property of invariance with respect to a transformation. For instance, if you swap the two halves of an image of a butterfly, or if you rotate a square by 90 degrees around its central axis, or if you shift a square grid (an infinitely long one) by one row, no change can be noticed following this operation. In fact, mirror symmetry, rotation symmetry, and translation symmetry are ubiquitous both in the natural and in the human-made environment. Another type of symmetry, which is particularly interesting, widespread in a variety of fields, and yet comparatively less well known, refers to transformations in

[3] Hermann Weyl, *Symmetry*, Princeton, Princeton University Press, 1952, p.5.
[4] Joe Rosen, 'Symmetry at the foundation of science and nature', *Symmetry*, 1, 2009, pp. 3-9.
[5] Klaus Meinzer, section "5.2. Symmetry as a Category of Cognition" in his book *Symmetries of Nature: A Handbook for Philosophy of Nature and Science*, Walter de Gruyter, New York, 1996.

scale. Scale invariance can be recognized, for instance, in clouds: zooming in or zooming out does not change the shape of the observed features; conversely, there is no way to tell the size of cloud features in a photograph, since clouds on different scales look very much the same. Scale symmetry can also be found much closer to us than clouds – they can even be seen on the shelves of grocery stores: broccoli or cauliflower with their patterns repeated over and over at different scales. Such isolated examples are unable to reveal the extent to which scale invariance impregnates our environment, but innumerable publications offer insights into their ubiquitous presence[6]. Scale symmetry brings us to the second key concept in this essay: scale.

Scale has a well-defined meaning in geography, being the ratio between the distance on a map and the corresponding distance on the ground. It is a number, which characterizes a tool we use to makes sense of our environment: the map. In this context, small scales refer to large areas captured on a map, while large scales belong to more detailed maps, which represent smaller geographical areas. Curiously enough, scale is not so sharply defined in other fields of science, where it does not denote well-defined numbers, but only qualitatively or sometimes comparatively suggests the size of the system that is addressed. Moreover, meanings of scale magnitude are interpreted in reverse, compared to geography: in physics, small scales and large scales imply small distances and large distances, respectively. This is the sense in which most other sciences also use the concept of scale, and this is how we will apply it here. However, when scale does not refer to a number (as it does in geography), one may ask what exactly defines it as an instrument. Indeed, only stating that the scale is small or large may not help us to specify useful information about the studied system, about our approach to the system, or about the interaction between the two. In fact, when one claims that scale is a tool for the exploration of natural systems one does not refer to scale itself, but

[6] The classical work on the subject is the one first written by the founder of the field of fractals, Benoit B. Mandelbrot: *Les objets fractals. Forme, hasard et dimension*, Paris, Flammarion, 1973. Many other books are dedicated to scale invariance aspects of natural patterns; the following are among those that offer insightful overviews: CC. Barton & P.R. La Pointe (eds.), *Fractals in the Earth Sciences*, New York, Springer, 1995; Donald L. Turcotte, *Fractals and Chaos in Geology and Geophysics*, Cambridge, Cambridge University Press, 1997; Bruce J. West, *Fractal Physiology and Chaos in Medicine*, Singapore, World Scientific Publishing, 2012; Perugini, D. & Kruhl J.H. (eds.), *Fractals and Dynamic Systems in Geoscience*, Basel, Springer, 2016.

rather to changes in scale and to the implications of such changes. It is in this sense that scale often represents a key instrument in pattern analysis, with its way of sweeping a wide range of scale values, like a wave covering “all” scales in one splash, thereby revealing in the system something that was not visible before the wave washed it all in. In other words, to turn scale into an exploration instrument we must include a mechanism capable of changing it, so we can consistently establish a relation between scale and some of the studied aspects of the explored system. Scale can be used both in a spatial and in a temporal context. In time, scale can be equivalent to the (variable) size of the window we open towards the environment. As is the case for its spatial applications, in the temporal context the assumption underlying its use is the idea that the way the studied system changes under transformations in window magnitude reflects relevant properties of the system. Since window size transformations are also subject to scrutiny in terms of the conservation of certain system parameters, scale is subsumed to the wider concept of symmetry. Although the paper emphasizes the wide range of spatial scales involved in various approaches to our natural surroundings, it does not include the full range of sizes currently accessible to scientific investigation: it does not descend into the microscopic realm, nor does it extend beyond the lengths involved in features of our planet. Similarly, the temporal scales we address are mostly confined to those involved in human experience, from seconds and minutes to years and decades.

2. Symmetry and scale in space

In a context possibly not intended to be interpreted as widely as it ended up being, Paul Valery stated that ”two dangers never cease threatening the world: order and disorder”[7]. It is tempting indeed to reflect on the general validity of this sentence, regardless of whether or not the author intended it to refer to the scientific worldview. Order and disorder are familiar concepts, widely used in everyday life. However, the goal of establishing definitions that would be scientifically rigorous, as well as useful in terms of our intuitive understanding of order and disorder, is quite elusive. The situation improves to some extent if we narrow down our approach to a specific context, such as orientation in a novel environment.

[7] Paul Valery, Crisis of the mind, first published in English, in *The Athenaeum* (London), 11 April 1919.

In this case, whether or not the surroundings are characterized by some degree of "order" is important.

Orientation in a featureless environment, or an environment in which features cannot be clearly distinguished from each other, is difficult or simply impossible. On a deeper level, this is what Mircea Eliade calls the "chaos of homogeneity"[8], referring to profane space experienced as an amorphous set of indistinguishable meaningless places. In order to pursue the objectives of this paper, we shall not dwell on the distinction between the sacred and the profane experience of space. We shall instead consider orientation and the assigning of meaning only in a secular – mainly scientific – sense. Imagine yourself in the middle of a snowstorm (or a sandstorm, for that matter): it is hard to figure out where you are and where you should go – you would not hesitate to associate such an environment with "disorder". The main reason why orientation becomes difficult this time is the absence of reference elements in space.

Let us turn now to the other extreme, in which order is guaranteed by pervasive symmetry, for example translation symmetry in two mutually perpendicular directions. Imagine yourself in the middle of rows of identical buildings, extending in front of you, behind you, to your left, and to your right: finding a specific building or defining your own position is, again, particularly challenging. We will use the phrase "symmetry saturation" to refer to the situation in which orientation is made difficult or prevented by extensive symmetry. As long as symmetry properties would hold to infinity as shown above, there would be no way to find reference elements in space to support your orientation. We know that it is an intermediate situation – between all-encompassing order and prevalent disorder – that would better serve our orientation purposes.

In a real, natural environment, in order to find our bearings, we are tempted to look for reference elements or landmarks, like a rock with a special, easy to recognize shape. This could represent a reference element for ourselves, but not for everybody else; however, what we often need is an orientation system that is shared with others. The process of using such landmarks for orientation might be fraught with difficulties. For example, unambiguously communicating the features of our landmark might be challenging. More importantly, potentially useful landmarks might not be available in the locations and with the spatial density we would need for our

[8] Mircea Eliade, *The Sacred and the Profane,* New York, Harcourt, Brace & World, 1959.

orientation. A uniform grid cast upon the landscape to be deciphered would thus offer a brilliant solution to the orientation dilemma. On a flat surface, a grid consisting of identical, square cells can work well, and indeed relatively small areas can be approximated with flat surfaces. Over large distances, the planetary shape changes the problem, and significantly so, but ingenious and useful solutions have been found for such challenges too. It is not the goal of this paper to outline the development of ideas meant to better grasp space; what we wish to highlight here is that symmetry can be effectively used as an order-providing ingredient to the study of a natural environment.

The fact that pervasive symmetry does not support orientation seems to be at odds with the fact that this is what we often do when we rigorously approach an environment, i.e. casting a net: a net so perfect that it guarantees symmetry (usually translation symmetry). To this end, all the cells must be truly indiscernible from each other. Having a net of identical cells covering the otherwise "disordered" landscape is supposed to be helpful for our orientation, and yet we have seen that pervasive symmetry cannot fulfill such a role. However, what we do for the purpose of orientation is not just to cast a net, but also to name, to number its cells. The moment the cells (or the threads in the net) are numbered, they start working for us, offering the reference elements we were searching for. However, the moment we take this step we also start distinguishing the elements in the net from each other: in other words, the symmetry condition vanishes. The grid elements cease being interchangeable. Each of them is unique. Rigorously speaking, orientation comes thus with the price of the destruction of symmetry. However, key symmetry properties of the network – and therefore of the network-environment ensemble – are still preserved; in fact, they remain essential for the way we handle not just orientation, but also a range of operations we apply to the analyzed environment: computation is strongly supported by the identicalness of the size of the cells and the ease with which their positions can be defined. While the "disorder" in the natural environment is difficult to tackle for one's orientation and hinders scientific analysis, it is made manageable with the help of symmetry features of the new instrument. It is an instrument that provides elements of order, without producing symmetry saturation, which would have obstructed our access to an intelligible environment. With such an instrument, we end up dealing not just with the landscape alone, but with an ensemble: landscape plus grid. While this looks like a modern procedure, it may be recognized as a recent form of a much older principle, already expressed in Plato's *Timaeus* (50-52)[9]: pure, eternal space is transformed, imbued with meaning, when "Ideas" impress themselves upon it.

Real-world features are usually seen as the product of two distinct categories of factors: laws and accidents (needless to say, not all scientists have been embracing this view, and hot debates have been dedicated to this subject). It is challenging to draw the line between the effects of these categories of factors when it comes to natural environments – consider, for example, topography[10]. Natural landscapes usually look so variable that one may not associate them with symmetry properties. And yet, a particular form of symmetry is often present and clearly expressed: scale symmetry. In fact, landscapes tend to look so similar when considered at different scales, that one is supposed to add an element of well-known size (such as a coin, a camera lens cap, a person) in geoscientific landscape photographs in order to indicate the spatial scale. This specific form of symmetry involves a specific form of order. One would expect such a pervasive form of order to strongly support spatial orientation. And yet, this does not have to be the case. The reason for this is, again, symmetry saturation, which can affect scale symmetry too. A world in which every fragment looks like every other fragment on a smaller or larger scale does not support orientation. For instance, as mentioned above, in an environment made of clouds, with similar shapes on all scales, it is impossible to tell how close or how far you are from some of the surrounding clouds; sizes cannot be figured out, and neither can distances.

So, when we remove the artificial reference, the symmetry-bearing square grid, from our topographic surface, we can discover that another form of symmetry lurks underneath. This may come as a surprise, as does the news that the underlying form of symmetry does not support orientation as expected. The reality is, however, that one is not completely lost in a natural environment when the artificial grid is absent: one can recognize and delimitate elements to be used for the design of meaningful paths through the landscape. The main reason why orientation is possible is the

[9] Plato, *Timaeus*, 50c/d, 51e-52b, in Plato, *Plato in Twelve Volumes*, Vol. 9, translated by W.R.M. Lamb, Cambridge, MA, Harvard University Press, 1925. See also chapter 3, especially notes 15 to 17, in Karl Popper, *The Open Society and Its Enemies*, Princeton, Princeton University Press, 1962.

[10] Donald L. Turcotte, *Fractals and Chaos in Geology and Geophysics*, Cambridge, Cambridge University Press, 1997.

fact that unlike ideal, mathematically defined topographic surfaces, real-world surfaces are not subject to symmetry saturation. In the case of scale symmetry, the avoidance of symmetry saturation is mainly ensured by the presence and properties of so-called "scaling regimes". The latter are intervals of scale over which the same scaling properties can be identified. There are two important aspects of natural scale-free systems which should be specified on this point: on one hand, scaling regimes are widespread in terms of system diversity and wide in terms of the range of scales that they cover; on the other hand, scaling regimes are always limited in size. The latter statement is not surprising (rather the opposite statement would have come as a shock), but its implications are significant. Let us briefly consider these two aspects of scaling regimes.

Scale invariance can be found for a very wide variety of patterns, from the microscopic scale to the regional and planetary scale and beyond, encompassing scale domains that are relevant to the structure of the universe[11]. In fact, scaling regimes often extend over wide ranges of scale, possibly spanning many orders of magnitude (up to 8 orders of magnitude for clouds[12]). While sometimes "pure" scale invariance (describable by one scaling exponent) can be found[13], numerous geophysical features – especially those involving fields – cannot be adequately described by one single exponent and a multifractal approach is required[14]. Even in the simple, monofractal case, more than one scaling regime can often be identified, and a shift from one scaling regime to another can be associated with the mechanisms dominating over each scale range; in other cases, a scaling interval can be adjacent to a scale range that is not characterized by scale invariance. The upper bound of scaling regimes is often imposed by the size of the system itself.

[11] Max Tegmark, *Our Mathematical Universe. My Quest for the Ultimate Nature of Reality*, London, Penguin Books, 2014, p. 108.

[12] Shaun Lovejoy, 'Scaling geocomplexity and remote sensing', in Quattrochi, D.A., Wentz, E., Siu-Ngan Lam, N. & Emerson, C.W. (eds.), *Integrating Scale in Remote Sensing and GIS*, Taylor & Francis, 2016.

[13] Scale invariance can also be present for geosystems subject to manifestations of anisotropy, which involve certain properties that change with scale range. *Ibidem.*

[14] Shaun Lovejoy, Daniel Schertzer, *The Weather and Climate. Emergent Laws and Multifractal Cascades*, Cambridge, Cambridge University Press, 2013.

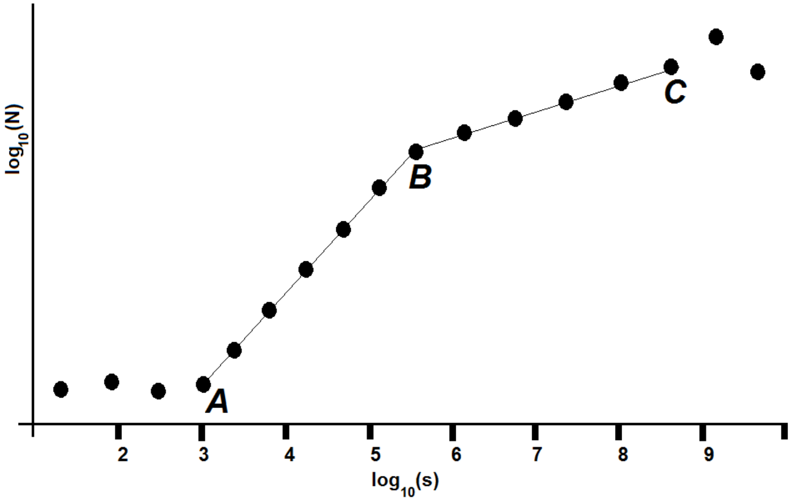

Figure 1. Graphical representation of two scaling regimes (from A to B and from B to C) flanked by intervals lacking scale invariance. Note the typically abrupt change in slope separating the two scaling regimes that meet in point B.

Since scaling regimes can be related to the processes that are responsible for the system configuration[15], their presence and especially their delimitation in scale space are often relevant to scientific investigation. The values of the limits of scaling regimes can be particularly important[16], sometimes even more so than those of the scaling exponents themselves. One is thus entitled to ask whether it is possible in practice to rigorously identify such boundary values of scaling regimes. The answer to this problem relies on an important property of scale invariance: in most cases, scaling regimes are sharply delimited from each other, with no gradual transition from one slope to another (figure 1). Scaling regime limits can therefore be usually identified with little or no ambiguity. In other words, in

[15] Peter Horne, Cristian Suteanu, Danika Van Proosdij, Greg Baker, 'Elevation-dependent multi-scale analysis of a complex inter-tidal zone', *Journal of Coastal Research* 29 (3), 2013, pp. 631-641.

[16] Luisa Liucci, Laura Melelli, Cristian Suteanu, 'Scale-invariance in the spatial development of landslides in the Umbria Region (Italy)', *Pure and Applied Geophysics*, 172 (7), 2015, pp. 1959-1973.

scale space, in which an “information flux” can be defined based on information invariance with respect to scale change[17], there is usually no transition, no inertia, no gradual transformation: departures from the principle “*natura non facit saltus*”, widely applicable in the macroscopic world, are here the rule rather than the exception.

The fact that scaling regimes are always limited in size represents one of the key differences between real-world patterns enjoying scaling properties on one hand, and mathematical scale-free features on the other hand. Most importantly though, scaling regime delimitation contributes to our capacity to distinguish various parts of the environment from each other, to find our orientation: instead of being immersed in a configuration consisting of similar shapes arising around us, with pieces of the environment of any size looking like pieces of any other size (as if we were climbing in an infinite tree with branches of all sizes branching out identically everywhere), we can tell where one type of configuration ends and another one begins. We can thus walk on a mountain slope without fear of confusing the ground that lies one step in front of us with a succession of peaks making us stumble and abysses in which we might fall at any moment. Things do not look alike everywhere, and not only because real macroscopic features always enjoy similarity rather than identity, but also because scale invariance always extends over a limited range of scales.

Finally, scale invariance manifestations are limited not only in scale range, but also in spatial extent: scaling properties of real-world features – such as topographic surfaces, for instance – change from one area or one region to another, depending on the processes involved in their transformation[18]. The two categories of limitation regarding scale symmetry – in terms of scale range and spatial extent – turn out to be key properties of the natural environment, which contribute to our understanding of our surroundings: they help us to distinguish features from each other and support our orientation.

3. Symmetry and scale in time

It should be stated right from the start that the temporal aspects of the environment addressed here are far away both from sub-atomic scales and from the speed of light. We consider the common situation in which we

[17] Cristian Suteanu, ‘A scale-space information flux approach to natural irregular patterns: methods and applications’, *Journal of Environmental Informatics*, 16 (2), 2010, pp. 57-69.
[18] Peter Horne, Cristian Suteanu, Danika Van Proosdij, Greg Baker, 2013, pp. 631-641.

use instruments to open exploratory windows towards our surroundings, and apparently get “instantaneous” snapshots. And yet those moments of exposure are long enough to make quantum uncertainty effects safely negligible. In other words, from this perspective our instruments are slow – but change in the studied systems is even slower: we study the world in slow motion.

We take samples of the system’s behaviour, we learn about the system’s state at those particular moments in time, but we do not know what happens in-between those snapshots – no matter how dense the temporal sampling may be. It is the resulting set of snapshots that we then spread out in front of us to figure out how the system is working: we study the world in slow motion with stroboscopic light.

When we explore the natural environment, we typically have access only to a limited part of the feature of interest. On the other hand, the studied feature is interconnected with other parts of the environment, and such interactions operate on different scales or across ranges of scale: we study the world in slow motion with stroboscopic light shone on a fragment of the system.

It is under such circumstances – based on sets of glimpses carefully collected and analyzed – that we expect to reconstruct the configuration of the system, the way it is related to other systems, as well as its dynamic behaviour, with the objective to also predict the future behaviour of the ensemble. Temporal approaches to the environment present their challenges.

Does this operating-in-the dark problem arise in the spatial context, too? Is space so much more different from time from this point of view? When we consider a spatial configuration, we see or think that we see the system as a continuous entity extending in front of us, and not as a set of discrete glimpses caught by instruments flashing in the night. And yet, when we perform measurements, we do apply a discrete grid, and the results reflect this discreteness and its characteristics – whether in space or in time. We may watch a river flow, or feel the wind: we perceive their flow as a continuous process, and yet when we start making measurements, we chop the flow into discrete bits, ending up with sets of numbers. Whether we measure air temperature with thermometers placed every twenty metres or trigger a temperature sensor every hour in the same place, the results are similar in terms of their discreteness. We may very well perceive the beauty of shapes in a topographic surface and handle its image as if it were a smooth blanket, a continuous entity, but when it comes to bringing it to our dissection table, we rely on a discrete set of points – nodes in a grid. Digital elevation models are becoming increasingly accurate, but their digital character always bears the fingerprint of the discrete net applied for their creation.

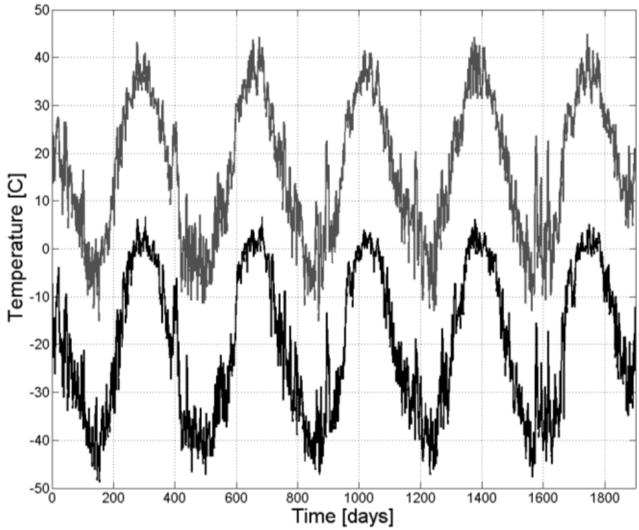

Figure 2. Excerpt of daily maximum (grey) and minimum (black) temperature record from Resolute (Nunavut, Canada).

Painfully aware of the limitations of our senses when it comes to absorbing the nature of reality, we generate devices designed to help us to see better, farther, and more clearly. We have learned indeed that what we perceive as direct access to reality might not be as direct as we think, that a veil might hang between ourselves and the feature we contemplate, and that this veil can also be a source of illusions. We have also learned that there is a price to pay for the replacement of the veil: we give up the appearance of continuity in our images. We have thus chosen to use instrument space-time grids instead: we traded the veil for an opaque screen with little holes in it. Holes in space. Holes in time. And the grid of holes is based on principles of symmetry.

Much like we apply grids to support our orientation in space, we use temporal grids to make sense of time. Unlike space though, time often has ubiquitous grids already embedded in its flow. Humans have been using such grids quite successfully, with daily and yearly patterns offering structure to their lives in innumerable ways. While non-embedded grids (external clock signals) are also used to study environmental features, we will focus here on the widely present and more interesting embedded grids in time. An example of such an embedded grid can be seen in figure 2. The graph shows seasonal oscillations in the succession of daily maximum and minimum air temperature values in the meteorological station of Resolute, Canada. The translation symmetry property in the temperature oscillation is strongly expressed in the graph, dominating the whole picture. Can we remove such deeply embedded periodicity, and if we can, what will be left behind?

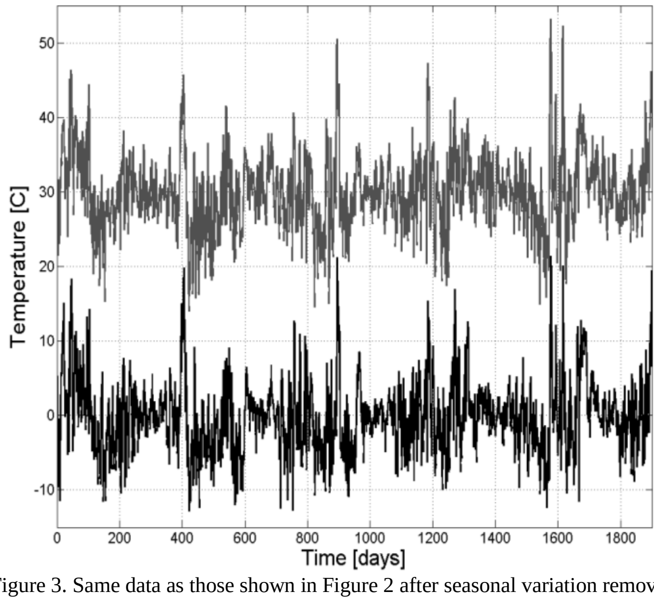

Figure 3. Same data as those shown in Figure 2 after seasonal variation removal.

We proceed by calculating the average temperature for each day of the year, i.e. taking the average of all dates of January 1st, of January 2nd, etc. in the temperature record. We then subtract the average value for each day of the year from each day in the record, e.g. we subtract the average for January 1st from all the January 1st dates in the dataset. What is left is a pattern like the one shown in figure 2. One can notice that the yearly periodicity can still be identified, since the variability of the pattern – the amplitude of what looks like noise oscillations superposed on a periodic variation in figure 2 – is stronger in the lower parts of each of the curves, i.e. during the winter. This increased variability occurring every year remains visible. These data look now like noisy oscillations around a zero average. We can take their integral (replacing each sample with the sum between itself and all the preceding samples) and produce the resulting graph – shown in figure 4. This graph is strikingly different from the other two. Its inset represents a fraction of the whole graph, and we can see that the patterns are characterized by similar variability.

The graphical form of the record from which the grid was removed suggests that the remaining pattern enjoys scale symmetry. To confirm this, we must proceed rigorously. There are numerous ways of assessing such scale invariant patterns. We will outline here a method that is straightforward to apply, leading to results that are suggestive and easy to interpret: Haar wavelets analysis[19]. The main idea is that we choose a window size – a "time scale" *s* – and calculate the average size *F* of the signal fluctuation for that particular scale. Then we repeatedly change the window size step by step and determine the fluctuation size in each case, after which we assess the relationship between the time scale *s* and the fluctuation size *F*.

If the time series is scale invariant over a certain range of scales, the relation between *s* and *F* will be a power law, and the resulting exponent of the power law will characterize the pattern.

To determine the average size of the fluctuation we apply the window of size *s* to the time series segment, divide the window in two, take the squares of the values in each of the two sub-windows, and subtract the result obtained for the first half from the one for the second half. We then move the window along the time series and repeat this operation, until we reach the end of the signal. We calculate the average fluctuation size based on all the results obtained for the window size *s*. This process is performed for each value of *s* we wish to assess. The final result of this procedure is illustrated in figure 5. We can see that not only is there a linear dependence

[19] Shaun Lovejoy, Daniel Schertzer, 'Haar wavelets, fluctuations and structure functions: convenient choices for geophysics', *Nonlinear Processes in Geophysics*, 19, 2012, pp. 513-527.

between *s* and *F* in logarithmic coordinates: this power law relationship extends over more than three orders of magnitude.

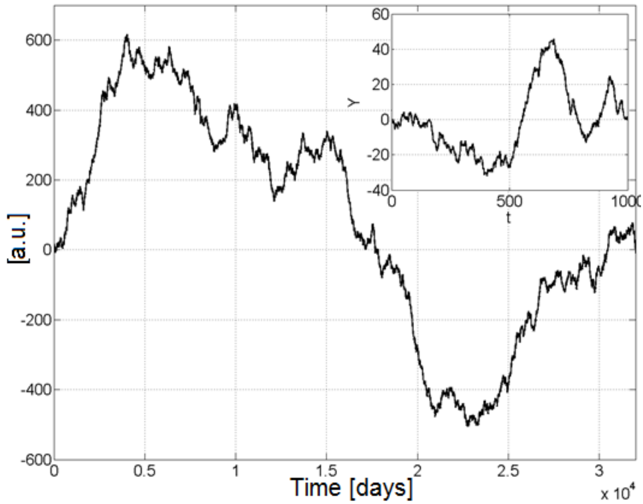

Figure 4. A segment of a daily temperature record after seasonal pattern removal and subsequent integration. The inset shows pattern preservation across scales: the fragment represents less than 1/30th of the entire graph.

We have thus started from a pattern dominated by a periodic grid, and after removing the grid we have found a pattern characterized by scale symmetry. Many other categories of temporal patterns with embedded grid periodicity behave in this way: there is scale invariance left behind when we extract the temporal grid; examples include natural processes subject to planetary regularities (such as river discharge, wind speed, and wave patterns), but also many other very different patterns, like those corresponding to physiologic parameters (e.g. heart rate, breathing, gait patterns)[20]. Such results are important both qualitatively (scale-free patterns reveal features of the physical mechanisms involved) and quantitatively, since the exponents reflect the variability expressed by the pattern on all scales over a certain scale range. For example, it was shown not only that the variability of air temperature patterns can be correlated with the distance

[20] Bruce J. West, *Fractal Physiology and Chaos in Medicine*, Singapore, World Scientific Publishing, 2012.

from the coast, but also that this correlation is expressed on scales ranging from weeks to decades[21].

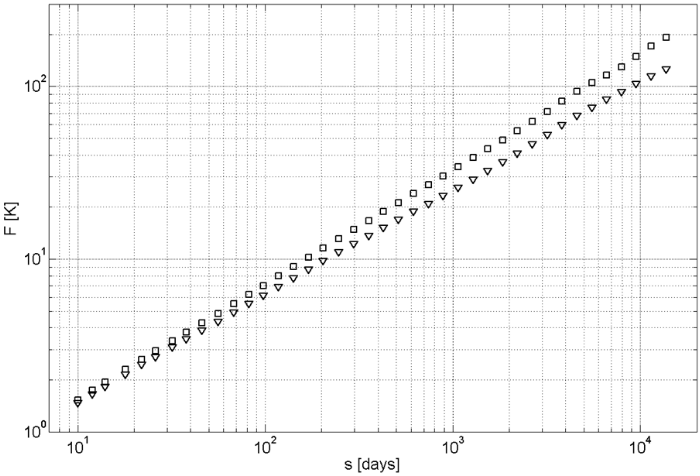

Figure 5. Outcome of the Haar wavelet analysis for daily temperature records: minimum temperature (squares) and maximum temperature (triangles). Size of the fluctuation F as a function of time scale s (in days). Note that the scaling regime persists over time scales from ten days to more than thirty years.

The properties of scaling aspects discussed for spatial patterns are valid in the case of temporal patterns as well. Scaling regimes are sharply separated from each other – in time too. They are limited in size. They are limited in temporal extent. Therefore, if we analyze patterns by testing scale symmetry at the same time with translation symmetry, we can detect pattern change. For example, it was shown that air temperature variability changes in time, such changes being expressed on a wide range of scales, but scale ranges that support scale invariance suffer transformations in their turn[22].

4. Conclusions

[21] Cristian Suteanu, 'Detrended fluctuation analysis of daily atmospheric surface temperature records in Atlantic Canada', *The Canadian Geographer*, 2, 2011, pp. 180–191.
[22] Cristian Suteanu, 'Statistical variability and persistence change in daily air temperature time series from high latitude Arctic stations', *Pure and Applied Geophysics*, 172, 2015, pp. 2057–2073.

Spatial and temporal aspects of the physical world have always been difficult to compare. On one hand, space and time seem to be so different from each other that comparisons do not even seem to be justified. On the other hand, similarities among them are so pervasive, that their blending has been perceived as deep and meaningful[23] and then explicitly applied in the realm of science: even long before Minkovskian spacetime was born, Lagrange called dynamics a four-dimension geometry[24]. It may be surprising that a glimpse into the distinction between them might be supported by considering the embedding of the grids in spatial vs. temporal patterns. Grids that are incorporated in the analyzed pattern are very different from those that we only interpose between the observer and the observed system. The latter are mostly applied in a spatial context (spatially embedded grids can hardly be found in the natural environment). In contrast, temporal patterns involve both kinds of grids: those deeply embedded, discussed in the preceding chapter, as well as an evolving set of “external” clock grids[25]. On this point, one may find that after all, we do not deal with temporal aspects per se when we analyze time series. Once the temporal flow of processes is turned into a succession of numbers, and eventually represented by a line in a graph, the entity we are facing is not temporal anymore, but spatial. This is a justified objection[26]. And yet, one may claim that the temporal character involved in the numbers or in the graph can still be perceived as such. Even typically spatial features such as decorative patterns running along walls can be read in a temporal key: words such as “periodic” referring to repeating shapes suggest a relationship with time. In fact, it is easy, almost tempting, for us to interpret a line running in some way along a horizontal axis as something occurring in time, which should perhaps not be surprising given the way our intuition is shaped and nourished through education. Perceiving temporal patterns – even when represented as numbers in time series or as graphs – in their actual temporal essence, rather than in their spatial disguise, is possible and, one would be tempted to think, quite common. It might thus be worth paying attention to our intuition concerning the distinction between patterns

[23] “…expansion and duration do mutually embrace and comprehend each other; every part of space being in every part of duration, and every part of duration in every part of expansion” – John Locke, *An Essay Concerning Human Understanding*, Book 2, Chapter XV-12.
[24] Joseph-Louis Lagrange, *Théorie des fonctions analytiques*, L’Imprimerie de la République, Paris, 1797.
[25] Robert Levine, *A Geography of Time*, New York, Basic Books, 1997.
[26] See on this point Bergson’s discussion on “spatialising time” in “*Time and Free Will”*.

in space and time, based on periodic grids being added and removed. Whether such intuitions should be seriously considered in the scientific endeavour, or, on the contrary, whether they should be disregarded or even suppressed, is a matter of debate. The author of this essay supports the former view: as long as intuitions are not applied to replace logical, evidence-supported steps in the development of scientific reasoning, but allowed to open new spaces and launch fresh questions, intuitions can be valuable.

Symmetry and scale, whether seen as distinct concepts, or addressed together as scale symmetry, offer valuable support to our effort to better understand our environment. David Deutsch, one of the main founders of the quantum theory of computation, suggests that scale invariance is not simply another property of certain physical systems, but has deeper meanings – it is intimately associated with knowledge[27]. In fact, while scale invariance may not be the main factor to be considered when one studies our environment, it may capture key ingredients of the intelligibility of reality.

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