FROM THE CATEGORY THEORY IN MATHEMATICS TO BIO-COSMOLOGY

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ABSTRACT

We argue that philosophy and mathematics could accomplish far more fruitful encounter with the Being, since by number it is possible to go to such an extent behind the reality (Pythagoras) and if the four causes of Aristotle would be (especially in the human sphere) over again actualized. Alain Badiou has already pointed that "mathematics is ontology," and now we have that the category theory in mathematics – having already covered other fields of this science – continues to find applications in a series of "non-traditional" domains of reality. In that correlation, philosophy could express too, its (primary) need for truth, justice, beauty, as well as for an overall development in the sense of human purposes – due to the undreamed power of the technological progress (say of hardware and software in informatics) today. In that manner, the philosophy of mathematics could radicalize its claims from the perspective of the slogan "One and All" of the first philosopher Thales and of such a (powerful) mathematical idiom in front of the reality of Being – this time, in the spirit of bio-Cosmology (neo-Aristotelism).

KEYWORDS: ontology, mathematics, category, final cause, bio-Cosmology.

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1. Mathematics as Ontology

From set theory to category theory

The original (Aristotelian) definition of science of ontology is a science of "being as such and the properties that belong to it"², just as the ancient philosophers (Pythagoras, Plato ...) looked at Being as to a whole of "one and many". But if we take Cantor's definition of *set*, we see that it was derived in sufficiently general mathematical, but epistemological (philosophical) terms too so as it makes possible the conclusions about Being and the knowledge about it. Namely, this definition is: "A set is gathering together into a whole of definite, distinct objects of our perception [*Anschauung*] and of our thought ..."³. In the 20th century, this explanation was extended, for example, to the attitude of Alain Badiou: "Mathematics is ontology". Certainly not in the sense the world is populated by mathematical beings, but that "during its entire history, mathematics expressed what is *sayable (dicible)* of being-as-such"⁴. This is because in the mathematical branch

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² Aristotle. "Metaphysics." *The Complete Works of Aristotle*. Vol. 2. Ed. Jonathan Barnes. Princeton, NJ, Princeton UP, 1984, 1003 *a*.

³ Georg Cantor. "Beiträge Zur Begründung Der Transfiniten Mengenlehre." *Mathematische Annalen* Vol.4 (1895), p. 46.

⁴ Badiou, Alain. L'être Et L'événement. Paris, Éditions du Seuil, 1988, p. 11.

of topology – which contains in its name the word *topos* (place) – plenty of results it achieves may immediately be "translated" into the language of ontology, and thus these results give the very expression of Being.

In other words, we have first that classical parts of mathematics: algebra, analysis, geometry, ... were interpreted in the framework of *set theory* which, on the line of generalization led – with some difficulties – to the concept of *category*, which by its name, almost directly refers to the importance of this word in Aristotle's doctrine of Being. Indeed, proceeding from the fact that both classical and non-classical theories are based (only) on the elements (sets) and relationships between them (functions) – as well as on some relations that accompany them – the words "element", "set" can easily be replaced (better: generalized) by the word "object," while the word "function" – by the word "morphism". The two mentioned relations would be: *identity* and *associativity*, etc., and so this meagre variety of concepts is found in the definition of the *category*, and one can show afterwards that category theory is a roughly identical (common) universe of reasoning in a series of mathematical branches, and not only in them. For instance, in topology the objects are topological spaces, and morphisms are continuous mappings between them, while in the linear algebra the vector spaces are the former and the linear mappings – the second ones and so on. In this way, now a set, or a multitude, is not determined by the properties of their elements, but by their overall relations with other sets, as the wholes.

In ontological terms, the case is the same with each individual: it becomes essentially determined by its relationship with other individuals from the *category* they belong together – as in the evolutionary sense it is a product of the *environment*. It's about, as F. Jedrzejewski says: "Not to develop an ontology in terms of the *logic of worlds*, but a topology of the universe, of space whose texture is so diverse to give more consistent meaning than the one done possible by a logical shell"⁵. Thus, the *topological concepts gain here a precedence over logical ones* – the notion of *environment* over the concept of *implication* – and so about Being and the One it is spoken out from the depths of topological objects and their morphisms, as they should order the appropriate logic too. Later, the concept of *topos* has received an exclusive place in their organization and classification of category–worlds, as well as their universal importance has facilitated the understanding of *identity* and *diversity*, of the *duality of Being and the One*, of their functionality, univocity... It would be a particular way back to the Heideggerian "forgetting of Being", other than that of addressing the idea (Plato), the substance (Aristotle), the monad (Leibniz) etc.

The notion of topos

By the notion of *topos* a strong intuition in mathematics is formalized, in so far as that we can test the properties of objects themselves, not only those from their relations with other objects. Just as it is in the *set theory*. A *topos* is a particular (Cartesian closed) category, with a special morphism (arrow) – the so-called *sub-object classifier* – making possible for each set to establish whether it is a subset of a given set or not. Its discovery in (about) 1960 not only contributed in solving problems of algebraic geometry, and elsewhere, or confirmed a highly synthetic power of this concept in the "unification" of mathematical knowledge, but turned out to be a powerful tool for the interpretation of Being too, in the methodological and epistemological terms.

We have after that the *topos structure* determines its appropriate logic, which is, in general, an intuitionistic one (here the principle of the excluded middle does not hold). But one finds that if the axiom of choice is valid on the *topos*, its logic becomes a classical one⁶. That is a far-reaching

⁵ Franck Jedrzejewski. Ontologie des Catégories. Paris, L'Harmattan, 2011, p. 6.

⁶ Razvan Diaconescu. "Axiom of Choice and Complementation," Proceedings of the American Mathematical Society,

significance of this proposition (theorem), since it states that the texture of a *topos* acquires its supremacy over its logic. And if the axiom of choice applies to it, then all infinite subsets have the structure of the One and Many, etc. Grothendieck showed equally that in topological spaces, open sets provide more information than the points they contain, or that the environment is "more important" than the belonging elements. Alain Badiou developed an ample reflection about Being and truth in his works *Being and Event* and *Logics of worlds*, in the closest connection with set (topological) terms, just on the trail of his thesis that back to Greeks the truth of Being is brought rather by *matheme* than by the Heideggerian *poieme*.

About the universal

As a notion, the "universal" is also subordinate to the concept of *categorical*, since it can be derived from it. The "category of all categories" is both "local" and "global", identical for all objects and morphisms. According to Jedrzejewski: "The universal comes from the categories and is not a form from outside, which would be materialized in particularities"⁷. As a logical concept, its negation is not the individual or the particular, but by the topological properties it possesses, it is rather recognizable on both sides: from natural to social sciences, as well as in everything surrounding us.

About duality

The principle of duality is posited at the basis of this world – both in what exists and in what becomes an expression of it. According to Jedrzejewski, "to understand the origin of dual forms represents an important stake for the science and philosophy too"⁸, because the dual concepts in the science of Being are *One* and *Many*, the *actual* and the *virtual* (which make real), *identity* and *differentiation*. It seems that this dual trait the Being has carried off on it since the Big Bang⁹, for, among others, it makes easier the understanding of one member of the dyad when we know the other one. In so far Badiou's slogan that mathematics is ontology, is being more truthful – since if mathematical and physical sciences bring the truth about the One and the One is dual with Many, as Being¹⁰ – the science of Being (ontology) would realize the relation of identity with them.

There are numerous examples of these concepts in physics, such as particles and waves, electric and magnetic fields, matter and antimatter, or Faraday and Ampere laws etc. In logic, those concepts are: *and* and *or*, *some* and *for every*, or De Morgan's laws, and in geometry – *point* and *right line* ("If a right line passes through two points", then "two lines intersect at the same point"), *sides* and *angles*. Among solids, the tetrahedron is dual to itself, so is the cube to an octahedron, a dodecahedron – to the icosahedrons, as are theorems of Pascal and Brianchon in projective geometry. And that is the case with many examples of mathematical objects, operations, assertions and theorems. Or: the objects have their co–objects, properties co–properties, diagrams co–diagrams and the like, so that it is just a way, for example, to classify the objects of a theory by the objects of another one.

The concept is of an exclusive importance in a number of theories and problems in mathematics (groups of Grothendieck–Galois etc.), in the string theory in physics, and so on. Let us say, finally, that this Janus' face of things and beings is recognized in epistemology too. Here the

^{51, 1975,} pp. 176-178.

⁷ Franck Jedrzejewski. Idem, p. 116.

⁸ Ibidem, p. 88.

⁹ Plato's dialogue *Timaeus* brings the myth of origins of the world from the ratio of the *Same* and *Many*.

¹⁰ For Badiou the member dual with the Being is "event" (*événement*), and for Sartre – "nothingness".

truth reveals a double character: it is *aletheia* as a truth of becoming and *adequatio*, as a truth of the One¹¹ etc. As to the categories, in the category theory, if you keep objects and change the direction of the arrows (sense of morphisms) you will obtain a category dual to the given one. The same may be said that for every graph there is a graph dual with it.

It should be said that many of couples of terms that "come together" (opposite, contradictory) are not mutually dual. Such is the case with the concepts of discrete and continuous, analytic and synthetic, necessary and contingent ... "What distinguishes the opposition from duality is that the latter is rather an identity, than a difference," says Jedrzejewski¹². So it belongs to dualities both an agreed and a "natural" existence (as in our examples of particles and waves, electric and magnetic fields etc.).

On functionality

If the Aristotelian notion of *relation* finds its analogue in the notion of *morphism* in category theory, it becomes generalized to the notion of *functor*, which brings the connection between objects of different categories – by transferring the properties of one of them in another category. A functor then, corresponds to the *analogy* in the intuitive area, which is as a methodological approach, otherwise, widely used in all domains of mind: from science, through religion, to literature (the concept of metaphor). For instance, when the parts of Being confirm their "identical" structure, or an "analogous" functionality, this only testifies about the unity of Being and our increased power to know it. So we talk about the supernatural world "according to the figure" of the world down, the same as the four types of statements in logic which Apuleius represents with spatial (planar) figure – namely by the "logical square". "Porphyry's tree" too during the Antiquity and the Middle Ages served to easier understanding, as Aristotle in *Nicomachean Ethics*, the moral terms compared with categories of mind in the same sense, and so on.

Finally, in physics, Feynman pointed out how "the same equations turn up in the study of several other phenomena: heat flow, distortions of stretched membrane, diffusion of neutrons, irrotational fluid flow, and uniform illumination of a plane"¹³, in order to conclude "that this is due to the unity of nature"¹⁴. Or to give the example of Newton's law of gravity (attraction of masses) and Coulomb's law (the attraction–rejection of electric charge) with the same expression and so on. Otherwise, the whole Being can be interpreted in terms of the One and Many – as those of farthest strongholds, or the common ground of all that arises and disappears in the world.

2. Category Theory as a Philosophy of Mathematics

And more than this: the category theory finds the way, in the same language, to transfer information about the objects, making it to our perceptions of them, fact testifying about a particular unity of subject and object too. Because: these perceptions do a category as well. This is a consequence of Yoneda's lemma, according to which each category C is isomorphic to the category of all functors from C to the category of sets **Set**. Then two objects are isomorphic too if the functors associated to them are isomorphic, so that between objects and "viewpoints" about them a relation of equivalence occurs. Therefore *the subject and the object are never separated in the knowledge*. And an object will be known from more aspects, if it may be observed in more

¹¹ Martin Heidegger, *On Time and Being*. NY, Harper and Row, 1972, p. 69.

¹² Franck Jedrzejewski, idem, p. 11.

¹³ After Penelope Maddy, "Ontological Commitment: Between Quine and Duhem." *Philosophical Perspectives* 10, 1996, p. 335.

¹⁴ Ibidem.

categories, and more functors are attributed to it. And so on.

We can continue to follow the language and principles of the *category theory* and try to bring them into a close relation with truths of the Being – which, in a philosophical key, Alain Badiou and Gilles Deleuze do – expressing rightly the hope that category theory's "synthetic power" in mathematics will be increasingly extended (applied) to other scientific disciplines, as to the Being alone. Moreover, especially the notion of *topos* favours the creation of "new mathematical worlds" with special properties, making possible too different categories and applications due to its provision. For example, every scientific theory has its "semantic core", linguistically expressed in different ways, determining *classifying topos* of the theory. Then, the theories are *Morita-equivalent* if they have the same classifying *topoi* – a term that, Olivia Caramello says in different ways"¹⁵. Finally, the properties of classifying *topoi* remaining invariant in the case of relation of equivalence are those being used to "transfer" the features of one *topos* theory into another.

And that some great parts of Being, according to the way they are ordered, yield indeed to a possible interpretation in terms of the mathematical theory, is suggested in O. Caramello's article under the title: *The Unification of Mathematics via Topos Theory*¹⁶. Some of these parts are, according to her: astronomy, genetics, psychology, linguistics, music. Namely, it belongs to a figure (albeit simplified) of the universe, in the first case, to be made by stars, around which rotate (finally) many planets. If we compare the latest ones with theories, their classifying *topos* would be the star, and they alone do an analogue of Morita-equivalent theories, established by the same *topos*. The fact that all planets perform the same type (of elliptical) orbits, is an invariant feature of star, to which correspond various examples of Morita-equivalent theories expressing the same property of classifying *topos*, in the *topos* theory. Here, for example, Kepler's third law is an invariant feature of the star-Sun, and it alone is a "bridge" to transfer information between two planets – because what is true in the case of some of them would have a counterpart in any other (as well as Kepler's law).

In the case of translation of a text from one language into another, afterwards, what has to remain invariant is a set of some of its abstract properties P which should be preserved (such as context, meaning, poeticity), taking into account the syntactic particularities of two languages as well. So in the case, say, of an expression e in the original language, we find first its "value" P_e in the context P, and then the corresponding value $P_{e'}$ of the expression in the context of language to which we translate. Here as a "bridge" appears the set P of properties to which we have alluded. Or in genetics, the human DNA plays a role of classifying *topos* in the *topos* theory. Because it does not depend on the human constitution, or age, as the classifying object is invariant too with respect to the manner in which the theory is expressed (axiomatization, etc.). In addition, they both have a role to point out the *similarities* and *differences*, transferring the knowledge from one area to another.

The *category theory*, therefore, testifies that an *intuitive reality* – ranging from nature to the human mind in their various places – yields an interpretation in its terms, and that the "knowledge" of one of them is transferable to another category, suggesting the possibility of systematic recognition of these circumstances and their studies, so that the solutions in one field to contribute to solving problems in another one. This, in a way of *construction* of adequate models etc., abreast with the technological progress contributing again and again to the possibility of "intrusion" in

¹⁵ Olivia Caramello. "Unifying Theory. Toposes as 'bridges'." General Overview.

www.oliviacaramello.com/Unification/ToposesBridges.html.

¹⁶ Olivia Caramello. The Unification of Mathematics Via Topos Theory. arXiv: math. CT/1006. 3930, 2010.

events, phenomena and processes, both in the external nature and in the human sphere, in order to express human needs in relation to them. Equally, therefore, at the macro and micro level, the man succeeds to predict phenomena, even if they take place almost chaotically (in meteorology, epidemiology, economics), as well as when they allow a very algorithmic solvability. So if a singular category–monad brings, for example, a category–model of preferably educated individuals, and be here foreseen the classifying *topos*, we would have the basis for the diffusion of patterns of an exemplary education in the case of an arbitrary number of (other) individuals in the same category.

Here is of importance the fact that we can attribute different "weights" (as quantities) in the human sphere to the subjective characteristics of human values and acts (moral, aesthetic and other norms) and then replace them by numbers. Thus, the reasoning in this domain receives, in addition, exactness, and it may be done, for example, in the framework of *fuzzy sets* and *fuzzy logic*. For a fuzzy set of a set sum up in itself the very qualitative labels of its elements, like the values of (fractions) between 0 and 1 - of 0 when it is absent, to 1 - when it occurs to the highest degree. Acknowledging the fact that everything in nature occurs in a way either of *juxtaposition*, or *integration* of elements–parts¹⁷ – for example, stones in a mosaic, in the first case, i.e., the hydrogen atoms and oxygen atom in a molecule of water in the other – we have the possibility, using set operations, to express what is new, as a whole of elements–parts, as well as to make choices desirable from some standpoint. And this: both in material and spiritual realm, still proceeding in terms of category theory.

Otherwise, it seems to us that the acquiring concepts about objects and beings are subjective in nature, not other than as a "set of their essential marks"¹⁸ from the standpoint of each of us. For if someone utters the words: table, house, tree, each of us will build (at least somewhat) different picture on them: and this would be confirmed if we put forward our own descriptions of these images. However, on the other hand, sentences are sets of words, and therefore some (subjective) value between 0 and 1 will equally pertain to statements. Therefore, in the choices we do in freedom, we are able to estimate the chance to realize them, in a sufficiently exact way – the way of mathematical models, counts. This would support the fulfilment of that ideal of Leibniz expressed by his formula: *Calculemus*!, in a case of disputes between the ideas, when it is not known which of them is more likely to succeed.

If one really has to choose, "unconditionally" and "alone" in freedom, the moral values for himself, would it not be desirable for him to know that search engine Google states (in October 2018, in English) 9.2 billions units for "good," and 653 millions for "evil," or 72.8 millions for "atheism" and 9.9 millions for "theism" etc? For no real experiment could count to such a large number of samples – with all the relativity in interpretation – "traditionally" so far. The search engine also says that people "mention" good ten times more than evil – otherwise, the number of them is measured in billions – and nine times more theism, than atheism. And so on. Finally, while we plead here for an eminent place which should belong to philosophy in an overall development of sciences and society as a whole – just as thoughtful ones – the test shows again that the index of Philosophy is 530 millions here, while it is 568 millions for Mathematics, 582 millions for Physics, 1.370 billion for Chemistry, or 925 millions for Biology. This tells a lot about the place of philosophy itself among the sciences and so on.

3. Bio-Cosmological view

¹⁷ Georges Chapouthier. "Mosaic Structures–a Working Hypothesis for the Complexity of Living Organisms". *E–logos: Electronic Journal for Philosophy*, 2009 p. 3.

¹⁸ Therefore, differently of Socrates' belief that the world can be thought only in necessary and general terms.

According to the holistic (organicist) standpoint of K. Khroutski, denoted by the phrase "bio-cosmology (neo-Aristotelism)", one considers the world as a *bio-cosmos* – just like Aristotle – "in which every (living) entity has its inherent place and destination in the one whole organic self-evolving cosmic world"¹⁹. These features already belonged to traditional Chinese medicine and to Hippocrates, so that many objections from the standpoint of modern methods of treatment today are because of the disregard of that fact. In biology, G. Chapouthier departs from the same viewpoint when he says: "If the general laws of the universe ... apply for living beings" we ask "what do the living beings teach us as universal rules or principles, which are likely to be found in the universe?"²⁰. As we recognize the words of Cornelia Guja in anthropology: "The human being as a system/interface may be considered a fundamental component of her/his human society and the nature/cosmos system as well, just like a hydrogen atom is the elementary constituent of matter under the material form"²¹. Denoted as *cosmism, universalism, organicism* ... at the ontological level, i.e. *intuitivism, life-cognition* at the epistemological one, or *anthropomorphism* in the axiological sense of the word, this worldview is worth to be both related to our problem of mathematics ontology correspondence, and considered its framework.

Like in the logic of included middle, the bio-cosmologies assume that in every life process (in appearing and disappearing) are identified *two polar opposites* and the *third* (fundamental, because it *integrates* the former). These are *spheres of reality*, including that of *ideas*, and within this ideal sphere, the ideas of *ideal conception* (understanding the causes, the circumstances and the consequences), *ways of realization* and *reasons to be* of facts, of ideas about them and of every part/moment of the *ideal-practical complex* are also included. On the other hand, if to the structure of Being belongs a whole of One and Many, they are precisely opposites, while inter-reality would do, again, the acts in which elements–parts enter into complex composites. *Triadicity* is the hallmark of the deductive systems too – with the *axioms* on one side and *theorems* on the other one, and finally with the proof "linking them". As for mathematics, the labels of triadicity are recognizable from one end to the other, and, as such, characterizing it to a highest degree, since, otherwise, it is said that the "whole mathematics is a theory of mapping". And a mapping is a whole of any two (independent) sets and of a defined relation between them. Just what essentially is repeated in the case of category theory, which also is based on objects and morphisms as relations among them.

Another main standpoint of bio-cosmology is a fourfold causality of all things, phenomena and processes in nature, just those coming to us from Aristotle. According to him, there are four kinds of causes: material, formal, efficient and final – as a furthest schematic "frame" through which passes the overall development in animate and inanimate nature. Once it was (in antiquity) determined by four proto–elements: earth, water, fire and air, as four natural forces are underlying in the universe: gravitational, electromagnetic, strong and weak nuclear forces etc. In Aristotle, everything is made of something, everything is created according to a form (model), everything is done by someone, and is intended to serve some purpose. The final cause in plants and animals is in their urge to preserve their own being, but also in striving to reproduce them, to replicate themselves. The same as the rational power in a human is a kind of a final cause in nature, if we

¹⁹ Konstantin Khroutski, "Bio-Cosmology–Rehabiliting Aristotle's Realistic Organicism and Recommending Russian Universal Cosmism: Response to Arthur Saniotis", *Eubios Journal of Asian and International Bioethics*, 18 (July 2008), p 98.

p 98. ²⁰ Georges Chapouthier. "Reflections on the Consequences of Bio-Cosmology in Modern Biology", *Bio-Cosmology–Neo-Aristotelism* vol. 1, No. 1 (Winter 2010), p. 93.

²¹ Cornelia Guja. "Bio-Cosmology and Informational Anthropology: Some Common Aspects", *E-logos: Electronic Journal for Philosophy*, 2008, p. 5.

consider that during the evolution the inorganic matter "has raised" to such a level of complexity, which is called "sense," "consciousness" etc. The biological composites, chemical compounds, mathematical structures also follow quadruple Aristotelian causality. For example, in the case of expression $x^2 + 1 = 0$, the final cause is that one for which the equations do exist in mathematics, while the other three types of causality are easily recognizable. And so on.

Therefore, the triadicity of all events and the fourfold causality appear as a largest (possible) frame, where we could really invest our belief that mathematical theories (like category theory) will succeed to incarnate in themselves our philosophical interest towards Being and man's place in it. It would be a (possible) task of the philosophy of mathematics.

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