A Methodological Remark Starting from Stephen Budiansky‘s *Journey to the Edge of Reason: The Life of Kurt Gödel*,

New York, W. W. Norton & Company, 2021. (Ana Bazac)

A popular book about a scientist is always interesting, for it concerns remarkable deeds which improved the human knowledge and thus, the telos, the reason-to-be of the human being and its civilisation. Accordingly, the life as such of the scientist, narrated by the popular book, is of absolutely secondary relevance since it does not explain the logic and uniqueness of the scientific construction. As already it is well-known, once created, a work has its own “life”, i.e., its own internal causes and significances. And since it’s about a scientist, just the life of his creation deserves to be displayed.

The honest, exact and beautiful documentary[1] of the life of Kurt Gödel, made by Stephen Budiansky, is nevertheless part of the tradition of books about the “romanticised life of...”, even in the form of non-fictional literature. But a scientist is not “Queen X”. To mix n details of the life of the scientist with some elements of his exploits is not useful for the average reader: neither from the standpoint of a deeper understanding of the social and psychological context of the activity of the scientist and nor from the viewpoint of this activity as such, or rather of the results of this activity. By speaking about “the life of...”, the writer transmits not only his analytical / critical cleverness but also various ideological values; consciously or directly pursued by the writer, but also showing through from the described tableaux as such. And although a strong tradition in the conceiving of the popular books is to create a fashion (or even a “market”) for the explicit and implicit ideological messages they promote, the profound reason of the popular books which flourished as a consequence of the Enlightenment movement in the Europe and America undergoing modernisation was just the propagation of knowledge by making it accessible to the general public. In this regard, the good popular books do not lower /reduce the quality of knowledge they “vulgarise”, they only make it accessible: and this requires a big technical expertise of writers.

The impressive documentation and ability to select the most significant aspects of the life of Kurt Gödel gives, however, a feeling of “incompleteness” and at the same time, of a “summary” of many issues that the reader would have liked to deepen.

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Every life is unique and has historical relevance, even if it is “happy”. We overthrow Hegel’s well-known opinion in his Lectures on the Philosophy of History. Every life

[1] With an impressive bibliography.

is plenty meaningful for us; we can grasp significances of every action / non-action, behaviour and their general structuring.

Every life is interesting; or it is not: for the understanding of the work. Kurt Gödel grew up as a curious child in a loving middle class family, he likened to learn, obtained his doctor in philosophy degree with a thesis in mathematics, had friends, a girlfriend he married; but also became psychologically ill – with paranoid delusions and psychosis, anxiety, obsessive compulsive disorders, phobias and schizophrenic manifestations. In a specialised popularising book, Pierre Cassou-Noguès, Les Démons de Gödel: Logique et folie, Paris, Seuil, 2007 proposed certain conjectures concerning the "unity" of Gödel's logic, philosophy and "madness": as a psychical background for his logic and philosophy. However, Gödel's logic was developed – in his young years – independently of his philosophical perplexities and influences and especially independently of his not yet serious health and psychical problems: because it concerned a special realm, mathematics, always realised in formal frameworks. Accordingly, we may conclude that the psychology (and even the ideology) of a certain author is not important when it is about scientific creations and works: because we know that there were different and opposed psychologies (and ideologies) of exceptional scientists working in the same period in the same domain. A psychological research of the hero, without discussing his scientific works but only labelling/classifying them according to the scientific opinions, is then a better outcome. And the description of the role of the imaginary in the work of scientists is already a specialised popularising book.

Stephen Budiansky is a historian, and thus he gave beautiful pages about the Gödel family's history and – something which is completely or partially missing from the other books about Gödel's life – the social contexts[2]. But even these could have been separate works, arousing the readers' interest precisely for the special problems treated in them. Or, in its current form, all of these aspects are truncated.

Gödel had reactions towards social ideologies and policies – like religion, racial discrimination, authoritarianism – and not only towards individual relationships.

And because the social ideologies and policies were too complicated, too far away from the logic of reason and too unknown, Gödel chose to be restrained from problems outside his field, he indubitably having democratic views. On the one hand, he assumed that all of these extra field problems did not have a priori legitimating, but they needed a thorough scientific investigation. On the other hand, what could he do but be silent, since he did not see – and he was not trained to see – logical solutions to complex social problems and since the offers of "specialized" interpretations of so many philosophers, sociologists and political scientists did not answer these problems than through unilateral criticism and never logically carried to the end?

During his stay in Europe, the dominant human model was that of the disciplined middle class and bureaucracy (see Heinrich Mann, Der Untertan, 1912/1918) towards which the liberal democrats seemed nec plus ultra. But even the close mathematician friends in the Vienna Circle, no matter how democrats and pacifists (and Gödel shared these views) were confused: because they lived in the triumphant era of capitalist economic modernisation and development – a kind of passive revolution, if we are allowed to use Gramsci's formula – and because they mixed theoretical liberal and utopian images

[2]I can'n stop mentioning that – unlike the other books about the "life" – Budiansky's book has described not only Vienna's and the Austro-Hungarian Empire's civilisation and cultural splendour seen from the perspective of the well-off middle class, but also its coexistence with its reverse in the conditions of life of the working class. See Stephen Budiansky, *Journey to the Edge of Reason*, pp. 39, 75, 76

But – for we are in a year of Olympiad – he did not mention the 1931 Workers' Olympiad in Vienna; see Gabriel Kuhn, Georg Spitaler, *These Stunning Photos Show How Workers Held Their Own Olympics*, 07.23.2021, `https://jacobinmag.com/2021/07/photography-socialist-workers-sports-international-red-vienna-olympiad` about society; not practical images related to the ardent needs of the working people, because their benevolent middle class humanism could not reach that. Their political impotency contributed to the final slip of Europe to the extreme right and could concretise only in their evasion (emigration) for survival. The (European) Belle Époque led to the WWI and later, the inertia of democratic protests did not stop the WWII[3]: because the evil did not consist only in racial discrimination, as some ones thought and still think[4].

In America, Gödel could manifest his – in Europe, social democratic, in the New World, liberal – interest for the American politics, but the mixture of hope and logical examination did not constitute a stimulus for his psychical balance, and concretised in the reduction of the discipline of creation. As many of his high intellectual friends, he was confused, choosing always the lesser evil: but within the logic of status quo.

The unity of the work and the state of mind of exceptionally endowed persons is a strong theme from Romanticism onwards. Nowadays we know that this relation is more complicated, and that people's philosophy or Weltanschauungen relate not only to their psychical state but also to the educational conditions suitable to absorb and understand philosophy. Gödel's confuse expression and metaphorical belief in angels and daemons has perhaps a link with his psychical troubles, but certainly it illustrates a confuse philosophy, based on an undisciplined philosophical training to judge criteria and outcomes. And in this respect, we should accept – and it's difficult not so much relating to past thinkers, but to present, living persons – that not everything a creator does is extraordinary and that, on the one hand, Gödel's philosophy remained at the level of suggestions – which imply n interpretations –[5], but that on the other hand, some of these suggestions prove to be fruitful.

The objective existence of mathematical truths – as Popper's "world 3" – and that this objectivity does not oppose their construction, was emphasised by Gödel[6]. Also: the fact that mathematical formalism is not the only source of mathematical validity[7]. Also: the fact that only the human reason can find new axioms, of a new kind, different from the old ones. However, somehow in the spirit of time – not in mathematics that was and is an "a priori science", but in philosophy and, generally, in ideology – he considered that with introspective ways and with phenomenology, we can grasp "other basic concepts hitherto unknown to us"[8]. Anyway, and letting aside phenomenology, "in the systematic establishment of the axioms of mathematics, new axioms, which do not follow by formal logic from those previously established, again and again become evident"; and that these axioms "are logically independent from the earlier ones": and, pay attention, just for this reason, "a machine cannot imitate"[9].

Gödel reflected both the ideological constraints of philosophy and its backward level from the standpoint of a clear language and expression, remaining at the level of metaphors and suggestions. His notes reflected both the influence and assumption of

[3]We can see a relative comparison in David Rosen, *A New Progressive Era?* July 8, 2021, `https://www.counterpunch.org/2021/07/08/a-new-progressive-era/`

[4]This type of reduction corresponds to the present liberal reduction of social problems to gender identity and sexual orientation. Philosophically, this reductionism – covered by its practice to mixing it with antiracism, in order to induce the confusion of their equivalence – promotes particularism and opposes the universalism of human reason. They are absolutely anti-Kantian.

[5]This philosophy was not scientific, as that of Carnap (who needed psychiatric services).

[6]See Kurt Gödel, "The modern development of the foundations of Mathematics in the light of Philosophy", (1961?), *Collected Works*, Volume III, Eds. Solomon Feferman, John W. Dawson, Jr., Warren Goldfarb, Charles Parsons, Robert M. Solovay, New York, Oxford, Oxford University Press, 1995, pp. 375-387.

[7]*Ibidem*, p. 381: "for reasonably comprehensive axioms of mathematics, it is impossible to carry out a proof of consistency merely by reflecting on the concrete combinations of symbols, without introducing more abstract elements".

[8]*Ibidem*, p. 383.

[9]*Ibidem*, p. 385.

idealism and the tendency to legitimate the "rightward philosophy" by the special situation of mathematics. His representation of objective existence of concepts and axioms can be likened to the vague, idealist, contradictory holism of the time. And the simplified views about materialism – considered to being opaque to freedom of randomness and to consciousness[10] – and idealism, did not allow seeing their unity. Also, his image about a valuable metaphysics demonstrating the objectivity of the world from some "primitive entities" analogous to the mathematical analysis of systems does not denote a deep philosophical understanding[11]: because philosophy is not reducible to the objectivity thesis and is historical, something that is different from mathematics. Finally, he had a simplified – and false – image about philosophy, or more exactly about its reason to be and its capacity. The deep intuition does not give the highest concepts and theories, and the logic and meanings of the world can be well emphasized even on the basis of rationalist demonstrations about the material intertwining.

Nevertheless, Gödel's philosophical insights were more subtle and less fit for rapid labelling than they were thought to be in different selective readings: he considered that "the middle" or the combination of materialism and idealism is the better answer[12] and that even in mathematics the certainty "is to be secured not by proving certain properties by a projection onto material systems – namely, the manipulation of physical symbols – but rather by cultivating (deepening) knowledge of the abstract concepts themselves which lead to the setting up of these mechanical systems, and further by seeking, according to the same procedures, to gain insights into the solvability, and the actual methods for the solutions, of all meaningful mathematical problems"[13]. And just because his starting point was mathematics, he considered that the process of understanding does not consist of giving explicit definitions for concepts, "since for that one obviously needs other undefinable abstract concepts and axioms holding for them", but of "a clarification of meaning"[14]. The necessity to use the phenomenological method and to understand Kant correctly was, for him, the important way to clarify the meaning of concepts: but for us this doesn't exclude to go further.

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Because Kurt Gödel's performance concerns logic and mathematics[15] (and especially mathematical logic[16]), the interest of the average reader is, obviously, to understand them: what formalism does mean, what a formal system does mean, what is the difference between the logical calculus[17] and the mathematical one, what the syntactic and semantic aspects of formal systems – and of their analysis – mean, what do mathematical truth, consistency[18], completeness[19] and incompleteness mean, what do proof and

[10]But do not forget: he wrote in a period when the "cold war" type dogmatism of both philosophical schools was strong.

[11]Gödel quoted in Budiansky, p. 270.

[12]Gödel, "The modern development of the foundations of Mathematics in the light of Philosophy", p. 381.

[13]*Ibidem*, p. 383.

[14]*Ibidem*.

[15]Certainly, Gödel had other mathematical contributions, besides his famous incompleteness theorems: in recursion theory, in the mathematical problem of intuitionism, and in set theory (independence results).

[16]Gödel's theorems belong to*mathematical logic*, because they applied logic to mathematics or, more precisely, they considered logic from a mathematical point of view, pursuing to understand the foundations of mathematics. And this made him a great logician.

[17]The calculus in a formal system supposes that there is always an algorithm of this calculus.

[18]Consistency means that the formal system in its complex structure is not contradictory and the deduction of results from this system does not lead to logical contradictions; simpler, that from the formal system one cannot deduce both a result and its opposite.

[19]Completeness depends on specific mathematical theories and refer to/reflects these specific con-

provability mean, and certainly what were Gödel's achievements, and not only through his famous theorems, what is the significance of constitution of new mathematical disciplines (as mathematical logic, but not only), what is the import of Gödel's theorems in the context of subsequent development of theorems solving the problems of consistency etc. And certainly, all of these can and is worth be related to the philosophical picture that preceded and evolved during the logical and mathematical endeavours. A "philosophy" of mathematical logic, namely, an explanation of its goals and tenets is much more interesting for the average reader[20]. Do not forget: it is about a popular book. Not the students in mathematics and logic are the target of a book about the life of Kurt Gödel[21], but just the non-expert readers. To give them truncated information, no matter how correct, about both the social and psychological context and the work[22] is not very beneficial.

For we speak about a popular book (or books) about scientists, the domain where they have created must be explained, no matter how rapidly. For example, the fact that in mathematics not only the rules of calculus, as it is in the common lay understanding, give the reason or correctness and efficacy of this discipline but also their foundations,

ditions, and Gödel Goedel has established (in his dissertation, 1929) the completeness of first-order predicate calculus, where, if every formula having a certain property can be derived in the system (using a theorem of the system), every true sentence in a model is provable; sentences are logically valid because they correspond to the model and are deduced on the basis of a finite formal deduction specific to it. Gödel considered that, since all the logically valid sentences are proven / provable, this system is complete.

About his completeness theorem, Gödel said: "The completeness theorem is indeed an almost trivial consequence of [*Skolem 1923b*]. However, the fact is that, at that time, nobody (including Skolem himself) drew this conclusion (neither from [*Skolem 1923b*] nor, as I did, from similar considerations of his own). . . . This blindness. . . of logicians is indeed surprising. But I think the explanation is not hard to find. It lies in a widespread lack, at that time, of the required epistemological attitude toward metamathematics and toward non-finitary reasoning. . . . The aforementioned easy inference from [*Skolem 1923b*] is definitely non-finitary, and so is any other completeness proof for the predicate calculus. Therefore these things escaped notice or were disregarded[135]", quoted in John W. Dawson Jr., *Logical Dilemmas: The Life and Work of Kurt Gödel*, Wellesley, MA, A. K. Peters, 1997, p. 58. The square brackets refer to Skolem's "Einige Bemerkungen zur axiomatischen Begründung der Mengenlehre". In *Matematiker kongressen i Helsingfors 4–7 Juli 1922*, Den femte skandinaviska matematikerkongressen, Redogörelse, 217–232. Helsinki: Akademiska Bokhandlen [A few remarks on the axiomatic justification of set theory].

[20]There already are such books. But after Hao Wang's *Popular Lectures on Mathematical Logic*. New York, Van Nostrand, 1981, other popular lectures covering the forty years after would have been useful.

[21]There already are such books, either insisting on specified aspects – like Hao Wang, *A Logical Journey. From Gödel to Philosophy*, Cambridge and London, The MIT Press, 1996; or Torkel Franzén, *Gödel's Theorem: An Incomplete Guide to Its Use and Abuse*, Wellesley, MA., A K Peters, 2005, whose clarity is once more emphasised by the mention of the life on only three pages; or the most recent and very undestandable Maria Hämeen-Anttila, *Gödel on Intuitionism and Constructive Foundations of Mathematics*, Doctoral Dissertation, Faculty of Arts, University of Helsinki, Unigrafia, 2020 – or developing chronologically, in "life" context, the professional accomplishments, like the very clear Dawson John W. Jr., *Logical Dilemmas: The Life and Work of Kurt Gödel*, Wellesley, MA, A. K. Peters, 1997. See also the review of Francisco Rodríguez-Consuegra, "Philosophy in Hao Wang's Conversations with Gödel. Review of Hao Wang, A Logical Journey. From Gödel to Philosophy", *Modern Logic*, Volume 8, Number 3 & 4 (May 2000–October 2001), pp. 137–152.

[22]Technical shortcomings can be found in Budiansky and Rebecca Goldstein, *Incompleteness: The Proof and Paradox of Kurt Gödel*, New York, London, W.W. Norton & Co, 2005. Goldstein's explanations contained mistakes as already shown by Solomon Feferman, "Provenly Unprovable", Review of *Incompleteness* by Rebecca Goldstein, *London Review of Books*, February 9, 2006, and Juliette Kennedy, Review, February 5, 2006, juliettekennedy2.pdf.

the axioms[23] - giving the frame of the formal systems[24] and their formalism (rules of inference) and underlying principles (or properties of mathematical objects) considered as the most general or productive – was a main concern of its professionals cannot be overlooked. Indeed, the great problem, more and more obvious in the second half of the 19th century was finding such axioms[25], and then, their checking in the "real life" of formal systems and rules of deduction, their verification including by confronting them with logical and theoretical paradoxes.

In this process, the need to understand and prove the correctness of formal systems and calculus following the given axioms, their consistency and the provability of their coherence led to the development of both mathematical logic and meta-mathematics. The first developed the reasoning from within the formal systems, while the second analyses mathematics from without, posing the problems of properties of mathematical objects, axioms and systems. Logic was a main tool in the axiomatisation of different mathematical disciplines, allowing inherent conjectures of mathematical theories and the speculation to reduce to one single axiom the foundation of a system of theorems. But logic was, too, a terrain where not only "the relation of mathematics to logic"', but also "fundamental questions of methodology, such as how quantifiers were to be construed, to what extent, if at all, non-constructive methods were justified, and whether there were important connections or distinctions to be made between syntactic and semantic notions"[26] were disputed. Actually, just this terrain favoured the constitution of meta-mathematics.

Is the above claim too pretentious? I think it is not. To supply the general public with highly accurate and readable information (about logic and mathematics, and especially about mathematical logic) means to leave behind the general traditional " life of..." and to substitute it with readable syntheses of high scientific theories. There are so many aspects which can and must be displayed[27] in order to increase the knowledge and scientific tools and worldviews of ordinary people that it is a pity to remain in old

[23] In mathematics, the axioms are considered intuitively *true*, and thus *truth* is given by the axioms and the necessary results of a solid calculus in agreement with axioms and a set of rules: or, philosophically speaking, with criteria that constitute the landmarks of research and theories developed within it. But in science, "truth" is provability, therefore the existence of proofs of the assumed theories.

Coherence proves the correspondence of the deductive steps evolved in the frame of the system with the axioms and sets of rules (of a certain formal system); differently put, the axioms and sets of rules are proven by this coherence of the deductive steps.

[24] Formal systems are *sets of axioms and rules of inference, allowing the generation of theorems*, thus sets of mathematical objects. The formal framework or the language of (formal) systems is given by languages with their alphabet and grammar, axioms, rules of inferences, theorems.

[25] If we want to explain philosophically: if the axioms are not contradictory to each other and do not lead to both true and non-true sentences, or to contradictory statements, the system has limits or is incomplete. In other words, Gödel's deployment of proofs always related to concrete formal systems, here to Peano's Arithmetic consistency – revealed the *limits* of systems of axioms: thus, and for both the alternative proving in other formal systems and the further progress of mathematics and mathematical logic, the axioms must be developed; this giving alternative systems which can prove with other means the consistency of Peano's Arithmetic.

More specifically: the *first* incompleteness theorem (1931) showed that, in the same consistent theory/formal system of Peano Arithmetic, there can be sentences which are neither provable nor disprovable; this would not meaning that the sentences cannot be proved in other formal systems. On the contrary, in principle they can / there are formal systems where these sentences can be proved. Nevertheless, Gödel showed that some arithmetical theorems cannot be proven within the *existent* formal systems containing Peano Arithmetic formalism. Therefore, it is always about the *limits of systems of axioms*, they are those which must be developed; not about the metaphysical impossibility to prove some truths. And the *second* incompleteness theorem demonstrated that the consistency of a certain type of formal system cannot be established with the means of the same formal system.

[26] John W. Dawson Jr., p. 48.

[27] See the beautiful Matthew Inglis, Andrew Aberdein, "Beauty Is Not Simplicity: An Analysis of Mathematicians' Proof Appraisals", *Philosophia Mathematica*, Vol. 23, Issue 1, 2014, pp. 87-109.

publishing clichés[28]. The general public has no time to address specialised scientific journals. Let's help them by bringing them closer to high quality scientific papers.

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We finish this review by mentioning some philosophical aspects related to mathematics, pointed out by the book or missing.

The ultimate philosophical nature of Gödel's famous theorems means that the idea of ultimate proof of a system as lying outside the system is valid because it is logical. People could see this logic many times and the philosophers could synthesise this common sense conclusion. But the point is to not remain at the level of intuitions or everyday proofs: it is to thoroughly demonstrate this idea.

The philosophical meanings of Gödel's exploits concern many aspects of their logic as such. The mathematical construction, its peculiarity towards the epistemological constructivism, the mathematical imagination and the criteria of analysis, the formal strictness of definitions, the limits of (formal) systems and any type of formalisation[29], involve logic[30]. Mathematical constructivism is the mathematical paradigm that considers the process of proving of the existence of mathematical objects, obviously as a result of working formalism in coherent systems And, while objectively, logic is the order of things, grasped by reason, as a discipline it is the normative science of reason, inherently simplifying it in ideal forms and giving criteria and norms to evaluate the validity and correctness of inferences. Just through the use of logic could mathematics pursuing the understanding of its foundations show that without the rigorous, inherently formal demonstration, the idea that in any system of transposing reality into a code of signs and significations, the last stage explanation is outside the respective theoretical system cannot be supported only at the level of philosophical (and common) intuitions.

However, Gödel's incompleteness theory was not conceived of as a proof of the finitude and limits of science / mathematics. The fact that a formal system – and the mathematical formal system can be a model for a scientific theory – does not arrive to its ultimate provability within its own boundaries is only an invitation to consider the expansion and nesting of systems (the systems of systems) and to scientific optimism. This scientific / mathematic optimism was promoted by David Hilbert and Hans Hahn[31], and was the life-long credo of Gödel. And though Gödel's idea about his theory and achievements was – in a lucid self-scrutiny – that "All of his contributions, he sadly observed, were of a negative kind—proving that something cannot be done, not what can be done"[32], actually they were methodological[33] and generated openness.

Not only that the "cannot" is fruitful (and science advances only through the excluding or negative proofs), but the theory drew attention at the same time on the fertility and the limits of (systems of) axioms (of truth sentences) in self-referential systems and

[28]These clichés issued in the era of the first industrial revolution, when first such books responded to the needs of the middle class and latter even to those of the lower classes. They popularised cognisance about the process of creation, about famous works, about the interdependence of social, psychological, philosophical and scientific sides of the human reality, even messaging to the readers that by hard-working every one can be a creator, something more important than to be a member of the haves. Anyway, these non-fiction biographies have – as we see today (but certainly I do not speak about Budiansky's book) – also the function to transmit dominant ideological meanings.

[29]Gödel's first theorem of incompleteness referred to the impossibility of proving the completeness of a certain particular system of axioms (Peano's formal theory of natural numbers qua elementary number theory), while the second theorem referred to the fact that the consistency of arithmetic cannot be established within the boundaries of the arithmetic system itself).

[30]But do not forget: logic applies to any knowledge.

[31]Stephen Budiansky, pp. 91, 95.

[32]*Idem*, p. 15.

[33]Is meta-mathematics not a kind of methodology?

the possibility / provability of necessary truths in modal logic, distinguishing them from contingent truths. Modal logic is related to mathematical induction in the sense that its operators help the constitution of finite chains of reasoning about expressible variables and quantifying sets and relations of natural numbers. However, "although mathematical induction is fully expressible in second-order arithmetic, the trouble is that the underlying logic (second-order logic) is not axiomatizable"[34].

There always are other systems – solving other problems (related to the intentions) – which have proven (internal) consistency but which, at their turn, are not complete. In a non-mathematical formulation that I don't know if appropriate, Gödel's demonstration that every axiomatizable consistent system in which all true sentences are provable is incomplete[35], is rather a proof of the historical trend of mutually generated mathematical theories and, obviously, of the union of mathematics and modal logic. Because, as Gödel observed, "that switching to higher-order systems of logic not only made it possible to prove propositions that are undecidable in a lower-order system, but also often dramatically shortened the length of the proof even for propositions that could be proved in the lower order system"[36].

Today we understand better the stakes of the former scientific theories without which, however, we cannot go on. The incompleteness theorem was considered too narrow because of its proof limited to a finite system[37]. Of course: but just this was its end, and by showing that "a static fixed FAS (finite axiomatic system) cannot work"[38], it suggested just the possibility to use the theorem in new, even opposed ways.

In his work, Gödel has developed some philosophical conclusions. First, they concerned the nature of mathematical truths that proved to be double: there are objective mathematical truths, not depending on 'any further hypothesis' and on any proof (whether this proof is possible or not), and subjective truths, as "humanly demonstrable" and conditioned by the axioms. Thus, even the system of axioms is not complete, but it contains both provable and unprovable truths. Then, they concerned the possibility to solve mathematical problems beyond the algorithmic procedures[39].

Other revealing philosophical conclusion was that formulated in a 1934 paper: in Menger's words, "that the consistency of each preceding system is provable in the successive systems; furthermore, that at every level there exist undecidable sentences that become decidable at higher levels", or otherwise put "in transition to logics of higher order, not only do previously unprovable propositions become provable, but many proofs already available become greatly abbreviated "[40].

Other philosophical conclusion is related to a speculative supposition of a complete set of axioms for the entire mathematics: if a system of axioms cannot prove all the true

[34]Raymond M. Smullyan, *Gödel's Incompleteness Theorems*, New York, Oxford, Oxford University Press, 1992, p. 113.

[35]But if, in other words, a system that is consistent cannot prove its own consistency is sound in a dialectic philosophy, in mathematics it is not. Because, as Gödel observed, a system/result is true when it is deduced from consistent axioms by consistent rules of inference. And if this is the case, there is no longer the need to prove it.

[36]In the words of Budiansky, p. 185.

[37]Gregory Chaitin, *Meta Math!: The Quest for Omega*, 2004, p. 23.

[38]*Ibidem.*

[39]Solomon Feferman, "Are There Absolutely Unsolvable Problems? Gödel's Dichotomy", *Philosophia Mathematica* Volume 14, Issue 2, 2006, pp. 1-19. A quote from Gödel (p. 12) is significant: "Turing gives an argument which is supposed to show that mental procedures cannot go beyond mechanical procedures. However, this argument is inconclusive. What Turing disregards completely is the fact that *mind, in its use, is not static, but constantly developing, i.e.*, we understand abstract terms more and more precisely as we go on using them . . . though at each stage the number and precision of the abstract terms at our disposal may be *finite*, both . . . may converge toward *infinity*".

[40]Karl Menger, *Reminiscences of the Vienna Circle and the Mathematical Colloquium*, Edited by Louise Golland, Brian McGuinness, and Abe Sklar. Dordrecht, Netherlands: Kluwer, 1994, p. 212.

sentences deduced within the system, would the construction of a complete set of axioms for the entire mathematics be possible? The problem is similar to the quest for a "theory of everything" in physics. The incompleteness of such a system and the impossibility to create it was explained by Chaitin: "for math to progress it would have to evolve over time, adding new concepts and new fundamental principles (axioms or postulates)"[41]. "The fundamental philosophical questions like the continuous versus the discrete or the limits of knowledge are never definitively solved"[42].

A fine philosophical conclusion resulted from strong relationship between mathematical formalism and its constraints and, on the other hand, the non-formal expression of the truth of principles or of "the significance of symbolic expressions"[43] as premises of the entire mathematical endeavour. If we consider these principles valid, there is no reason to not accept all the mathematical truths and proofs based on them. A kind of reciprocal is the understanding of the limits of mathematical[44] concepts: for instance, consistency as the key or most important proof/basis of the truthfulness of the mathematical objects and theories. However, the consistency as such is not enough to attest any theory / rather one cannot attest any theory only on the basis of consistency of the formal system[45].

What is interesting is that Gödel tried to mix and at the same time to surpass the divergences between the schools of mathematical foundation whose friendly and unfriendly dialogue led to so many relevant discoveries in the mathematical field in the first half of the 20th century. He did not reject formalism, even in its constructive form, while showing that logic helps to understand that the mathematical constructions and truths are not aleatory. They are necessary, thus objective. Beyond the discussion about Gödel's Platonism, it is about the avant la lettre assumption of the "world 3" of Popper[46]: the human theories etc., once created by man, become a world distinct from him, and that can be treated and judged independently of its constructors[47]. The truth of theories, for-

[41] Gregory Chaitin, in *It's not All in the Numbers: Gregory Chaitin Explains Gödel's mathematical Complexities*, 2012, `https://www.simplycharly.com/read/interviews/its-all-in-the-numbers-gregory-chaitin-explains-kurt-godel-mathematical-complexities/`

[42] Gregory Chaitin, *Meta Math!: The Quest for Omega*, 2004, p. 10.

[43] L. Susan Stebbing, Postulational Systems and *Principia Mathematica*, originally published as Appendix in *A Modern Introduction to Logic* (1931), Third edition, Methuen, 1942.

[44] For a lay person like me, all of these aspects of mathematics send to questions related to science and knowledge.

[45] Torkel Franzén, "The Popular Impact of Gödel's Incompleteness Theorem", *Notices of the AMS*, Vol. 53, Number 4, 2006, pp. 440-443 (443).

[46] This idea that the logic of the created spiritual things is that which gives them legitimacy and not the fact that they were conceived of by the human mind, appeared in Benedictus de Spinoza, *Ethics* (Ethics Demonstrated in Geometric Order…), (1677), in *The Collected Works of Spinoza*, Edited and Translated by Edwin Curley, Volume I, Princeton, New Jersey: Princeton University Press, 1985, V, 23, Note to proof, p. 608. In Latin, "Mentis enim oculi quibus res videt observatque, sunt ipsæ demonstrationes", `https://la.wikisource.org/wiki/Ethica/Pars_quinta_-_De_potentia_intellectus_seu_de_libertate_humana`

[47] Letting aside Gödel's image of the objectivity of spiritual creations as if they would prove the omnipotence of a trans-mundane being, his view was, nevertheless, consistent. His Platonism was simply a manner to express the objectivity of spiritual creations.

"He maintained, for example, that because a mathematician cannot 'create the validity of . . . theorems . . .at his will,' mathematical activity 'shows very little of the freedom a creator should enjoy.' On the contrary, he argued, 'what any theorem does is . . . to restrict [that] freedom,' and whatever restricts the freedom of creation "must evidently exist independently' of it... He agreed that 'a mathematical proposition says nothing about the physical or psychical reality existing in space and time, because it is true already owing to the meaning of the terms occurring in it.' But he rejected the contention that 'the meaning of th[ose] terms . . . [is] something man-made, consisting merely in semantical conventions.' Instead, he reaffirmed his Platonistic view that 'concepts form an objective reality of their own, which we cannot create or change, but only perceive and describe.' The meaning of mathematical statements thus inheres in what they say about relations among concepts", John W. Dawson Jr., p. 199.

malism, conjectures and developments of infinite values does depend only on the internal logic or consistency of the theoretical constructions. But on this line, truth is necessary even for the demonstrations of formalist consistency.

Logic and mathematics are distinct, but they are intertwining and their history – at least, in the 20th century – is a race where the relay was taken over conjointly by mathematicians and logicians. Gödel was an exponent of this race and the development of fundamental positions and theories in both sciences was very clearly emphasised in the book of Dawson Jr, but less clear in Budiansky's book. (Both Budiansky and Goldstein leaned only on the famous incompleteness theorems). Gödel's exploits in the founding of mathematical logic can be understood only in concert with the whole of other discoveries before and after them, just because they were answers to the problems debated in the community of mathematicians.

I think that just this concert and logic of creations should be the main topic of general books about scientists. And: much less the overwhelming life details and the details of personal professional relations. In Budiansky and the books cited until now these details show a careful reading of mathematical papers and deciphering of the shorthanded manuscripts. Nevertheless, the chronological story of details of personal professional relations is not always welcome in order to transmit the logic of creation. Perhaps a future chronological analysis of all manuscripts of Gödel will better show both the richness of his professional endeavour and the philosophical confusion that has not impaired his mathematical and logical depth, but is interesting to know from a didactic standpoint.

Popularising mathematics and logic is important. It's not the place here to elaborate, but it's certainly necessary to say that the popularisation of mathematics is difficult. It depends on the level reached in the mathematical and logical research, or from which the popularizer of these disciplines starts. Thus, it must follow Hilbert's advice: "A mathematical theory is not to be considered complete until you have made it so clear that you can explain it to the first man whom you meet on the street"[48]. For this reason, on the one hand, to be a popularizer of math without being a mathematician is hard. On the other hand, for a mathematician to transpose into lay explications, rather using words than formulas, is not very comfortable. When explaining clearly, people clarify also their thoughts: by speaking, transposing into words some intuitions or unfolding some not so clear reasoning, they arrive to know. But a mathematician already knows: thus he doesn't feel the need to say by words that which he not only understands very well but is better explained in the specific mathematical language. So, only those mathematicians who consider popularising their discipline a duty, do it. From this standpoint, both the books of Budiansky and Goldstein have shortcomings.

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