HISTORY OF FORMALISM: FROM ARISTOTLE TO GÖDEL

Robert DJIDJIAN[1]

r.djidjian@gmail.com

ABSTRACT:

Using formal means for developing scientific theories became a tradition from the times of Aristotle's *Analytics*. Ernst Schröder built the complete algebraic theory of inferences by the end of the 19th century. The idea of a complete formalization emerged as a way for eliminating paradoxes in foundations of mathematics that Bertrand Russell has revealed at the very start of the 20th century. Bertrand Russell and Alfred North Whitehead developed the first completely formalized theory in the three volumes of *Principia Mathematica* (1910 - 1913). David Hilbert enhanced the formation of metatheoretical approach to axiomatic theories by his call for proving the consistency of mathematics by using only finitary means. All of a sudden, in this atmosphere of steady axiomatic studies, a young mathematical genius Kurt Gödel published his famous theorem, which proved the incompleteness of a formal arithmetic system. Gödel's theorem raised a huge wave of metatheoretical studies of formal systems. His main instrument, called Gödel's numbering, was a special type of self-referential expressions that caused paradoxes just in foundations of mathematics. An aspect of Gödel's approach, that may raise discussions, is the formalization of metalogic itself, which actually may eliminate the idea of metatheory.

KEYWORDS: Aristotle, Ernst Schröder, *Principia Mathematica*, David Hilbert, Kurt Gödel, metatheory.

Contents

1. Introduction

At present days of logical science, two main areas of research can be distinguished – research on the theory of proof in the framework of mathematical logic and development of the methodology and logic of scientific research. The foundations of the first direction were laid by Aristotle's two *Analytics.* Eventually, the theory of proof was crowned with Gödel's famous theorem on formalized theories (Gödel 1931). This history took about 24 centuries and raised a huge wave of publications on different aspects of formalized theories (Kleene 1952, Goldstein 2006, Smith 2007, Raatikainen 2022).

Aristotle's theory of deductive inferences, *syllogisms* in terms of Aristotle, is presented in *Prior Analytics*. The core of the theory is developed in the first seven chapters of book one of *Prior Analytics* by revealing all valid modes of inferences from two propositions having a subject-predicate structure. All the remaining 95 % text of the *Prior Analytics* is about inferences containing modalities and false premises.

Aristotle's syllogistics is considered a perfect theory in the sense that it presents the proofs for all valid inferences from any two types of categorical (subject-predicate) propositions (judgments). Aristotelian strict proofs of valid modii make the impression that his syllogistic theory

[1] Professor, physics, philosophy, Yerevan, Armenian State Pedagogical University

is an example of absolute truth of the same level as that of Euclid's geometry. The great critic of dogmatic theories, Immanuel Kant, had the highest opinion of the Aristotle's logical theory: "formal logic was not able to advance a single step (since Aristotle) and is thus to all appearance a closed and complete body of doctrine" (Kant, 2004, p. VIII).

Yet, under the apparent influence of mathematical sciences, in particular, of algebraic equations, there was formed a strong belief that syllogistic inferences can be performed in algebraic manner. Gottfried Leibniz dreamed creating a *calculus ratiocinator* that would make all arguments "as tangible as those of the Mathematicians, so that we can find our error at a glance, and when there are disputes among persons, we can simply say: Let us calculate" (Wiener, 1951). Valuable attempts in this direction made George Boole in his *The Laws of Thought* (1854) and Stanley Jevons in his book *The Principles of Science* (1879). The algebraic approach to the theory of inferences and proof got its perfect and detailed formulation in the three volumes of Ernst Schröder's *Vorlesungen über die Algebra der Logik* (Lectures on the Algebra of Logic) (Schröder 1890-1905).

By the end of the 19^{th} century and the first two decades of the 20^{th} century a new system of symbolic logic emerged, nowadays considered as the dominant theory of inferences – the mathematical logic. This new direction was generated by research in the field of the foundations of mathematics. The pioneer here can be considered Gottlob Frege, who published in 1879 the book *Begriffsschrift* (Terms writing, i.e. calculus of concepts). The system of symbolic designation of inferences in *Begriffsschrift* was so unsuccessful that this work of Frege did not receive due attention. However, the system developed by Giuseppe Peano played a significant role in the formation of the symbolic language of mathematical logic (Kennedy, 1980). Anyway, the fate of Frege's two-volume work *Grundgesetze der Arithmetik* (1893/1903), devoted to the substantiation of number theory, turned out to be more successful. Frege's approach was copied in Bertrand Russell's *Principles of Mathematics* (1903) and developed further in three voluminous volumes of *Principia Mathematica* by Russell and his former university teacher Alfred North Whitehead (vol. 1 – in 1910, vol.2 – in 1912, vol. 3 – in 1913). Further events unfolded around the concept of formalization of axiomatic theories, set out in the famous article by the young mathematician Kurt Gödel (1931). But before we get into Gödel's concept of formalization, we need to be clear about the notion of formal theory.

As shown in the history of science (of mathematics), the first step towards the formalization of a theory is the introduction of letters and symbols to describe objects and formulate the statements of the theory. The central moment of the theoretical representation of the doctrine is the axiomatic representation of the theory. In the axiomatic representation of the theory, the basic statements of the theory (axioms) and all statements derived from them and the corresponding definitions using the rules of inference (legitimate inference schemes) are considered true.

The crown of scientific knowledge is the ***proof.*** Scientific research is an incessant search for proving the truth of an important statement for the answer to the problem under study. In this aspect, the axiomatic construction of the theory has a fundamental advantage. Opponents of the theory need to be able to present a fact that refutes any of the axioms or is inconsistent with any definition. This task is very difficult, because authors of theories, putting forward their axioms and definitions, had considered all significant facts.

By the end of the 19^{th} century, research began on the axiomatic construction of the most basic mathematical teachings – set theory and number theory. And suddenly, like a bolt from the blue, paradoxes were discovered in the very foundations of mathematics. As a natural reaction, the idea of a more rigorous formulation of axiomatic theories appeared, and then the concepts of formal and formalized theories became widespread.

2. Aristotle’s formal logic

Aristotle (384–322 BCE) created a significant number of fundamental sciences of ancient times like Logic, Psychology, Physics, Philosophy, Rhetoric, Cosmology, and many others. The logical works of Aristotle – *Categories*, *On Interpretation*, *Prior Analytics*, *Posterior Analytics*, *Topics*, *On Sophistical Refutations* – later were called *Organon* meaning "instrument, tool, organ" of cognition. Actually, *Analytics* contain the Aristotelian theory of logical proof: *Prior Analytics* presenting the theory of syllogistic inferences, while *Posterior Analytics* – the general concept of proof, including the Aristotle’s teaching of definition.

Aristotle was very scrupulous in presenting the teachings and views of his predecessors. He called colleagues to sum up what is known by this point to be able adding a new result to this heritage. Yet, in regard to the science of logic, Aristotle emphasized his priority. He pointed out that when he was developing the science of inferences “it is not true to say that present it had already been partly elaborated and partly not; nay, it did not exist at all …regarding reasoning we had absolutely no earlier work to quote but were for a long time labouring at tentative researches” (*On sophistical refutations* 155; 34, 183b, 184a).

The striking thing about Aristotle's *Analytics* is that there is not a single case known in the history of science when a theoretical concept was created without predecessors, as if from scratch, and yet was created as complete perfection. There were three factors that could facilitate the creation of *Analytics*. First, there was a certain atmosphere of analysis and research in the Socratic dialogues with great skill presented in the writings of Plato. The Armenian ancient philosopher David Anhacht (David Invincible, 6th century CE) pointed out in his *Commentary on Aristotle's Prior Analytics* that Plato did not need Aristotle’s theory of proof but rather Aristotle took from Plato’s works the seeds of his logical teaching (Tophchian, 2010, ch.4). One could be surprised by David Anhacht’s remark since Plato had not written any work dealing with problems of logic in general or the theory of proof in particular. David Anhacht’s words should not be taken literally. The main idea of his remark is contained in the term “seeds”. True, there are no elements of logical *theory* in Plato’s works. But his dialogs are full of rational discussions and attempts to find out definitions of various concepts.

The second factor could be the appearance of sophists, before and during Aristotle’s time. They composed a new social group of citizens able to teach youngsters in a wide range of subjects, with particular emphasis on skill in public debates. Due to everyday educational practice with young people, sophists eventually created rationalistic climate of thought on questions about morality, religion, and politics. So that by the days of Aristotle and the sophists, the “collective intellect” of the nation has risen to such a level of strength that a Greek individual felt himself able to solve any problem (e.g. Aristotle) and prove any statement be it true or false (the sophists).

From the days of Aristotle’s *Metaphysics*, there was quite a satisfactory understanding of the essence of scientific knowledge. Scientific knowledge, in contrast to the opinions of people, had to have strict **proof**. By the 4th century BCE the deliberations on the reliability of sensual data and rational judgments brought to the formation of the school of philosophical Skepticism. The main statement of Pyrrhonist skepticism asserted that knowledge of things is impossible. Skeptics have maintained for several centuries an ideological confrontation with dogmatism presented by the very influential philosophical school of Stoics. Yet this criticism of the positions of opponents had a very specific feature: neither the Academics nor the Stoics had a more or less satisfactory conception of truth. Both disputing camps did not use the fundamental definition of the truth, suggested in Aristotle’s books on the first philosophy, and continued their confrontation even not mentioning Aristotle’s valuable conception of scientific knowledge.

Compared to Skeptics, ancient sophists presented the opposite pole that reflected the unique degree of intellectual development when human mind first succeeded in proving own opinions.

Should not they think that they were really wise, or *sophoi* in old Greek? Especially, if we would consider that sophists were ready to teach Athenian youngsters becoming wise as were they themselves. But sophists demanded payments for their lessons for which they were criticized in Socratic dialogues. Aristotle wrote a special work – *Sophistical refutations* – where he revealed the ways by which sophists pretended being able to prove true and false statements equally.

3. Algebra of logic and Mathematical logic

The system of reasoning created by Aristotle from two categorical judgments was considered as perfect that for more than two millennia, from the 4th century BCE and until the beginning of the 17th century, the theory of categorical syllogism existed without significant losses and gains. But already from the middle of the 17th century, the idea of an algebraic representation of the theory of inference was born. Many attempts in this direction were made by Gottfried Leibniz and his followers Johann Lambert, Julius Plücker and others. Interesting results in the algebraic representation of inferences were obtained in the middle of the 19th century by George Boole, Augustus de Morgan and Stanley Jevons. By the end of the 19th century, the system of algebraic inference theory was exhaustively presented in Ernst Schröder's three-volume work *Vorlesungen uber Die Algebra der Logik* (Schröder, 1890-1905).

From the point of view of the history studies in foundations of mathematics, the paradox of the set of all "normal" sets, discovered by Bertrand Russell in 1903, is considered the first and most significant paradox. From the point of view of common sense, specific objects and their sets belong to completely different "worlds", as if they were opposite to each other. The set of books is not a book. Objects are separate entities, while sets consider their collections (groups). These two heterogeneous types of concepts are connected using the concept of *property*. Usually, a set is defined as a collection of objects that have a given property. In the case of the set of books, this unifying property is that of having pages. At the same time, it is considered natural that the attraction of a certain property for the formation of a set implies the formation of something new, different from the objects themselves, the elements of the set. A set of books can form a library - a new object with its own socially significant functions.

In the light of the said, posing the question of a set that can be its own element is something unexpected and strange. Indeed, are there such "anomalous" sets that they themselves are their own elements? Which set-forming property can ensure that the resulting set has this same property? The question is not easy, and requires accurate deliberations. Namely, this "anomalous" characteristic feature should be a property that would in a hidden form designate both a certain set of available objects and the property itself. In the field of research in the foundations of mathematics, such a set is "the set of all sets". Since it is the set of *all* sets, it will also include itself as its element. At the same time, there is nothing problematic in the concept of "the set of all sets". There is no paradox, hence, there is no problem.

It was B. Russell who pointed out that already a derivation from the concept "the set of all sets", namely, the concept of "the set of all *normal* sets", generates a paradox, a logical contradiction. To reveal the paradox, we divide all sets into normal sets (not containing itself as an element) and anomalous sets (containing itself as an element). Now it is easy to show that *the set N of all normal sets* is paradoxical.

If *N is a normal* set, then it must satisfy the condition of not being a member of itself and thus it is not the set of *all* normal sets, which is in contradiction with its definition.

If *N* is not a *normal* set (is an anomalous set), it must be a member of itself by definition. But the set N of all *normal* sets is composed of only *normal* sets and as such cannot be a member of itself, which is a contradiction too.

Thus, we came to a paradox – in the both possible cases we have a logical contradiction (compare Irvine & Deutsch 2021).

It is quite natural that for several decades' attempts to resolve Russell's paradox were carried out within the framework of the problems and categories of mathematics, in particular, the set theory. Indeed, as it is clear from the review article on Russell's paradox in the Stanford Encyclopedia of Philosophy (Irvine & Deutsch 2021), by that time, mathematicians were not inclined to see a connection between paradoxes in the foundations of mathematics and classical paradoxes, primarily with the Liar paradox. Russell himself saw the solution to the paradox of the set of all normal sets in his "type theory", according to which the formation of a set of sets (predicates from predicates) should be limited. On this way of eliminating specific contradictions, the mathematicians Zermelo, Frenkel, Skolem, Neumann already in the first decades of the last century built axiomatic set theories, free from contradictions like that of Russell's paradox.

However, such a partial solution of the problem for many mathematicians did not seem to be satisfactory. Generation after generation, mathematicians found it natural to build theories for all times, in the likeness of Euclid's geometry. Quite in the spirit of this need, the famous mathematician of the last century, David Hilbert, put forward the idea of proving the consistency of mathematics using only convincing, finitary means. This would free all mathematicians from the uncomfortable feeling that a new paradox might arise again in some area of mathematical knowledge.

Another significant result of research on the foundations of mathematics and the construction of axiomatic theories has been the increased attention to the rigor of the language of mathematical theories. As a result, an ever-increasing tradition has emerged for constructing formalized theories and studying their properties such as completeness and decidability in the frame of non-formal metamathematics (metalogic).

4. Gödel’s theorem under scrutiny

Gödel’s incompleteness theorem (formed by two related theorems published in the same article in 1931) of the formalized arithmetic (Peano Arithmetic) had a major impact on the modern researchers in mathematics, logic and philosophy. Actually, Gödel's 1931 article has determined the philosophy and ideology of all subsequent studies on the foundations of mathematics. There arose an important wave of publications on the consistency and completeness of formalized systems (Smullyan, 1991; Franzén, 2005) and on the philosophical interpretation of Gödel's theorem (Rucker, 1995; Wang, 1997; Feferman, 2011).

By definition, formal (or formalized) theory is said to be consistent if no formal proof can be carried in that theory for a formula *A* and at the same time for its negation *~A*. The consistency of mathematics became a central problem of studies in foundations of mathematics due to *Hilbert’s Program*. The main idea of this approach was quite simple – to prove mathematics consistency using only finite means. Hilbert with his colleagues and some other researchers got certain results regarding concrete axiomatic theories of number theory. In contrast to Hilbert's standing, Gödel's theorem on the incompleteness of formalized arithmetic proved that Hilbert's program was unrealizable: it followed from Gödel's theorem that by means of a given formalized theory it is impossible to prove its own consistency (Gödel's second theorem).

The general idea of Gödel’s proof is quite clear – to build some formula ***A*** *unresolvable* in the system of (Peano) formal arithmetic. The problem of *resolvability* (*Entscheidungsproblem*) had interested mathematicians due to *Grundzüge der Theoretischen Logik* (Fundamentals of Theoretical Logic), published by David Hilbert and Wilhelm Ackermann (1928). According to the definition, if ***A*** is an *unresolvable* formula then both ***A*** and non-***A*** (the negation of ***A***) are unprovable. (We cannot say which of them is true). On the other hand, according to the law of excluded middle we have “***A*** or non-***A*”,** one of these two should be true. These means that there is a truth (***A*** or non-***A***) that is unprovable in the system of formal arithmetic. In short, Gödel's theorem proved that the system of that formal arithmetic is *incomplete*. It showed that formalism is depending on the given axioms of the given system, it is not a simple set of deduction/inference rules and by eventually adding new axioms the given system is still incomplete. Meaning, the Peano Arithmetic axiomatic system is/must be consistent – otherwise it is not useful – but it is not necessarily complete, and one cannot demonstrate that it is both consistent and complete, one cannot prove within the PA neither a true statement about PA consistency (first theorem) and nor that there is not a statement that asserts both *A* and ~*A*.

However, besides the idea and ingenious demonstration of Gödel’s theorems there are some aspects which may rise some discussions.

First, Gödel proved his theorem by constructing in formalized arithmetic some formula **G** that is true but unprovable in Peano Arithmetic/from the axioms of *this* system, the result being the inconsistency of this arithmetic system. And here’s the puzzling detail: when interpreted meaningfully, formula **G** means: “Formula **G** states that formula **G** is unprovable”. In the formula **G** only one predicate is used – “provability” – as possible to be formalized in arithmetic. This fact unambiguously implies that the formula **G** belongs to the theory of proof, part of the same formalized system, but not of the same arithmetic theory. Thus, considering this aspect, it turns out that Gödel’s theorem proves also the incompleteness of Gödel’s formalized proof theory, besides that of formalized arithmetic.

Secondly, Gödel built his system of formalized arithmetic, including his fundamentally important formula, with the help of a special numbering invented by him and called *Gödel numbering*. Briefly, the essence of the Gödel numbering is as follows: each predicate, each symbol, each formula, and each expression of the formal language of arithmetic is assigned a distinct number, due to which the formalized system becomes arithmetized. It was with the help of the special numbering invented by him that Gödel was able to construct his formula **G**, which asserts its unprovability. Expressions that state something about themselves are called self-referential. Very close to Gödel’s self-referential formula is the well-known from antiquity paradoxical formulation "The proposition **L** states that the proposition **L** is false" (the Liar's paradox). The paradoxical statement **L** generates a contradiction – both the statement **L** and its negation ~**L** turn out to be provable. Moreover, since the middle of the last century, mathematicians have recognized that all the paradoxes identified in the foundations of mathematics arise precisely because of the self-referentiality of the expressions used. Accordingly, there is a serious possibility of the emergence of a new paradox – a paradox at the level of the meta demonstration – generated by Gödel's self-referential formula.

Alfred Tarski proved in 1933 a theorem according to which in the first order formal arithmetic the concept of truth is *not definable* using the expressive means that formal arithmetic affords. If the formal arithmetic would contain a predicate **Tr** that in its informal interpretation means “to be **True”** then one could build with the help of Gödel numbering a “liar” paradox type formula $S \leftrightarrow \neg True(g(S))$ where g is Gödel’s number of the formula *S*. The interpretation of the formula *S* in the informal arithmetic means “*S* says S is false” – an exact expression of the “liar” paradox (Tarski, 1983).

Yet, revealing a “liar” type paradox in the system of Gödel’s arithmetized metalogic, Alfred Tarski suggested a very mild conclusion: truth is undefinable in formal languages (Tarski, 1983; Hodges, 2018). Actually, Alfred Tarski has revealed that formal theory with arithmetized metalogic is contradictory in the sense that one can build in this system a formula that in its informal interpretation expresses “liar” paradox $S \leftrightarrow \neg True(g(S))$. According to metalogic approach, revealing a paradox in a formal system one should conclude that this formal system is contradictory (Baaz et al, 2011; Fereiros, 2008).

We would like to mention also that even the mild interpretation of Tarski’s theorem as of undefinability of the truth in formal systems is essentially damaging the concept of formal (arithmetized) metalogic. Not having the predicate truth in a formal metalogic (it is present only in the arithmetic formal systems), one cannot judge either on completeness, or the consistency of a formal system. Then what is the use of such a metalogic system?

What was Gödel's reaction to the difficulties revealed by Tarski's theorem? There was no single comment on Tarski's undefinability theorem in any of Gödel's published articles (Wang, 1997).

A. Tarski proved his theorem using Gödel numbering. Until there will be suggested a proof for Tarski's undefinability theorem without using Gödel numbering the opponents of self-referential sentences would insist that the undefinability of truth is caused by Gödel numbering.

5. Non-correct definitions as the main source of paradoxes

The whole problem of consistency, "perfection" of an axiomatic theory nests in its definitions. It is enough to use one unfortunate (fraught with paradox) notion in a fundamental theory for generating a corresponding paradox and starting panic in this science. For some reason, scientists and analysts do not notice that the paradox concerns only this concept and relevant judgments, while theory as a whole does not "care" about this paradox. We mean that specialists continue to study and develop this theory, being convinced that sooner or later researchers will be able to resolve the revealed paradox. For example, Russell himself, who discovered the paradox in connection with the concept of the "set of all sets" in 1903, already in 1910 proposed in the first volume of the *Principia Mathematica* a "theory of types" to exclude the possibility of the appearance of the said paradox precisely by limiting the applicability of the concept "set of all sets".

An axiomatic theory is built from three main parts: a small group of initial statements of the theory – axioms; a small group of logical rules for deriving consequences from available statements (premises); and an unlimited group of definitions of notions formulated as the theory unfolds.

It is implicitly assumed that axioms are either self-evident or that they have earned their high status of a basic statement by the fact that many important statements of the theory are deduced with their participation. Yet, let us assume that there is a doubt about certain axiom of a sufficiently developed theory as of a potential source of a paradox. But since we are talking about a fairly developed theory, the suspected axiom, among other axioms, had multiple cases of use in the derivation of new statements of the theory. This means that the defectiveness of the considered axiom should have manifested itself many times. The history of sciences demonstrates that theories face only single cases of paradoxes. This proves that the axioms of a sufficiently developed theory should not be considered as the cause for the appearance of a paradox in this theory.

It must be borne in mind that the "immunity" of the axioms of proven theories in relation to paradoxes does not extend to their resistance to new, previously unknown facts. The appearance of principally new facts that contradict this axiom means only the fallacy, and not the internal inconsistency of this axiom. The new observational data obtained with the help of telescopes, combined with the laws of Newtonian mechanics, refuted the postulate of geocentrism and the entire Aristotelian model of the universe. However, the postulate of geocentrism was not self-contradictory and did not lead to paradoxes. Conversely, the expression "This statement says it is false" and similar expressions such as Russell's paradox are self-contradictory and generate paradoxes independently of any facts.

Similarly, rules of logical inferences are also a small group of rules. Since we are considering a sufficiently developed axiomatic theory, each of the inference rules has already been repeatedly used in the proofs of the theorems of this theory. If some logical rule of inference were so defective that it could generate a paradox, then dealing with a highly developed theory and the intensely use of its inference rules, many paradoxes should have arisen, while paradoxes in the history of scientific theories are single cases only.

The axiomatic method of constructing of a theory, namely, especially, the unambiguous definition of all the concepts of a given theory, also excludes the possibility of a logical contradiction due to the ambiguity of the natural language used. Just the fact of defining each notion of an axiomatic theory eliminates the ambiguity of the language used. This means that the criticism of the use of natural languages in axiomatic theories is, in fact, pointless. It is the obligatory definition of each term (notion) in the axiomatic formulation of the theory that eliminates the very possibility of errors and contradictions due to the use of a natural language.

The situation with paradoxes is not saved by the formalization of the theory, the transition from carrying out proofs in natural language to purely formal transformations of the statements of the theory, written down as a purely symbolic expression (a sequence of letters and other signs). The very procedure of rewriting the meaningful definitions of a non-formal theory into the symbolic language of a formalized theory is performed mechanically, following the rules of the given formal theory. At the same time, *if there is some inadequate (unspecified) definition of a term in the original non-formal theory, then this defect of the definition will be accurately reproduced in the corresponding symbolic notation of the formalized theory*. In this case, a definition is so "bad" that in the original non-formal theory it implies a *truth value* paradox, so the same paradox will reappear also in the formalized theory as a *provability* paradox.

This means that the formalization of a *non-formal* axiomatic theory cannot give anything positive aimed to securing its consistency. The axioms have to be restated.

In the case of Gödel's arithmetic formalization, the latter studies of formalized systems raised the problem of the means to give useful solutions either in the aspect of eliminating the appearance of local paradoxes, or in the aspect of the possibility of proving the consistency of mathematical theories. We believe that the lack of content of formalized theories cannot significantly damage the development of mathematical sciences, but it can disorientate young researchers toward neglecting aspects of definition in the formalization of axioms and theorems.

Conclusions

The above analysis has revealed three main concepts of formalism:

A. Formalism as an approach for eliminating paradoxes in foundations of mathematics,

B. Formalism as a program for consistency proof by vary means,

C. Formalism as a concept of total arithmetization of a formal theory.

All of these options were developed in the name of creating an impeccable, "ideal" version of the axiomatic theory, but apparently, the axiomatic construction of the theory is not subject to further improvement. In the axiomatic theory, problems and paradoxes arise mainly due to the unsuccessful, inadequate definition of a notion.

The first approach presumed that by eliminating natural language from the means of scientific research and argumentation will eliminate the very source of paradoxes. Actually, the elimination of natural language was carried out by rewriting expressions in natural language into the symbolic language of the formalized theory, following its predetermined rules. As shown above, if there is some inadequate (disproportionate) definition in the original content theory, then this defect will also be reproduced in the corresponding symbolic notation. That is, the formalization of the axiomatic theory by the elimination of natural language and symbolization of a theory cannot give anything positive in terms of the emergence of contradictions and paradoxes.

Hilbert's research program of proving mathematics consistency by finitary methods presumes that researchers are able to find out in some way the indicators of any statement provable in mathematics, which is completely non-realistic, and Gödel demonstrated this.

According to Tarski's theorem, in any interpretation of a formal system using the predicate "to be true", we will unavoidably express the liar paradox. But in science it is impossible to abandon the truth. Without the truth, there could be no scientific knowledge. Judging about a formalized system by its “purely” formal (arithmetized) meta-logic is an attempt of judging about the chains of symbols using the chains of equally meaningless symbols.

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