

Landscapes in Logic Vol. I. Contemporary Logic and Computing,

Adrian Rezuş, editor; College Publications, London, 2020.

(Cristian Calude¹)

This volume aims to illustrate the interplay between the contemporary work in logic and computing and the mainstream mathematics². It is divided into two parts: Selected Topics in Contemporary Logic and Advances in Computing³.

The first part includes eight chapters

The chapter "The Development of Decidability Proofs" by K. Bimbó presents the evolution of decidability proofs for various propositional logics based on sequent calculi, from the first one – for intuitionist logic – to some of the latest results. The common thread is the essential use of the Curry Kripke technique.

"Non-Distributive Logics: From Semantics To Meaning" by W. Conradie, A. Palmigiano, C. Robinson and N. Wijnberg discusses a line of research in the relational (non topological) semantics of non-distributive logics.

The chapter "Bounded Functional Interpretation with an Abstract Type" by P. Engrácia and F. Ferreira presents, in a classical framework, the bounded functional interpretation with an abstract type and its main result.

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²Project, Adrian Rezuş (ed.) «Contemporary Logic and Computing Science», College Publications, London 2020. Goal: The book consists of invited and contributed papers illustrating recent research trends in logic and computing science, edited by me during the period September 2019 - June 2020. It is focussed on the following topics: A) Proof theory, lambda calculus, type-theory, and constructive mathematics, B) Recursion theory, computability, computational complexity, and applications of logic in computer science, C) Current research on classical and non-classical logics, D) Philosophy of mathematics. The Project is part of a larger editorial enterprise, currently in progress at College Publications, London. Date: 1 September 2018 - 1 July 2020. <https://www.researchgate.net/project/Adrian-Rezused-Contemporary-Logic-and-Computing-Science-College-Publications-London-2020>.

³The division of the volume in two sections – topics in 'logic' vs topics in 'computing' – is more or less conventional. Some contributions are focussed on historical and technical details meant to put in perspective the impact of the work of some outstanding mathematicians and philosophers on the contemporary research in logic and computing science. Some other papers, also with a historical flavour, were supposed to evidence punctual methods of research and specific concepts or topics, as, e.g., decidability, computability, randomness, and computational or descriptive complexity. In general, the papers were intended as specific surveys of results. Other volumes – to be issued subsequently in the same series – will hopefully delineate aspects of the contemporary logic landscape that have not been illustrated here. The intended audience of the book includes graduate students in mathematical logic, foundations of mathematics, and computing science, as well as philosophers, mathematicians, and, possibly, other scientists interested in the recent research on logic and computing. July 2020, 978-1-84890-340-1. <https://www.collegepublications.co.uk/LiL/?00001>

L. Humberstone discusses in "Twins in Logic – Identical and Otherwise" what Łukasiewicz metaphorically calls twins connectives in some logic, that is, when they behave 'in the same way' according to that logic. The presentation includes looser and stricter ways to read this metaphor.

A. Iorgulescu presents a detailed view of a world of twenty two algebras of logic in "Algebras of Logic vs. Algebras"

"Relevant Logics: From Semantics To Proof Systems" by H. Kurokawa and S. Negri presents in a uniform manner axiomatic presentations of a wide family of relevant logics as well as their Routley-Meyer semantics (based on ternary accessibility relations).

"Sobociński's Nachlaß" by V. F. Rickey presents unpublished results on Leśniewski's systems in B. Sobociński's papers. Items that the author was unable to place chronologically are listed at the end of the paper.

W. Veldman's paper "Treading in Brouwer's footsteps" presents the work that has been done in intuitionistic mathematics by J. J. de Jongh and some of his students in the period 1963–1985. The paper briefly treats the philosophy of intuitionistic mathematics, and then continues with some results concerning the continuity of real functions, Dedekind-infinite sets, the continuum hypothesis, the perfect set theorem, and the completeness of predicate logic, descriptive set theory and Ramsey's Theorem.

The second part on Advances in Computing includes thirteen chapters

A. Böhm, M. Böhm, E. Böhm, M. Dezani-Ciancaglini, F. Manfredini, N. P. Böhm present in "Corrado Böhm. The λ -adventure" a glimpse of Corrado Böhm, "scientific genius, but also [the] man".

C. T. Chong discusses in "The Reverse Mathematics of Ramsey's Theorem for Pairs" the combinatorial principle RT_2^2 derived from Ramsey's Theorem for pairs and presents its proof-theoretic strength within the framework of reverse mathematics.

The chapter "Ramsey Theory on Infinite Structures and the Method of Strong Coding Trees" by N. Dobrinen discusses some recent trends in Ramsey theory on infinite structures and possible future directions for applications.

R. Downey chapter "Randomness and Computation" presents a selection of results in Algorithmic Information Theory, an area which uses computational methods to define and study randomness of individual objects. Interactions with computability theory and applications in information theory, effective dimension, randomness amplification, and analysis and ergodic theory are covered, but not the relevance to quantum theory and computing.

O. Finkel and D. Lecomte chapter "Descriptive Set Theory and Ω -Powers of Finitary Languages" surveys some recent results that link Descriptive Set Theory and ω -powers.

W. Gasarch's chapter "Low, Superlow, and Superduperlow Sets" ("Exposition of a Known But Not Well-Known Result") presents the beautiful "proofs of Jockusch and Stephan" for the existence of these strange mathematical objects. And, indeed, they look beautiful even for the reviewer who is not a fan of this exoteric chapter of incomputability.

W. Gasarch, E. Metz, Y. Shen, Z. Xu, S. Zbarsky present an intriguing result about the size of the smallest non-deterministic finite automaton recognising cofinite unary languages and related open questions in "Small NFA's for Cofinite Unary Languages".

C. Gassner's "An introduction to a model of abstract computation: the BSS-RAM model" gives a detailed presentation of BSS RAM's for the sequential computation over first-order structures, including important properties like universality.

N. Greenberg's chapter "Two applications of admissible computability" presents two applications of admissible computability in the formalisms of Kripke and Platek to higher

randomness and computability of uncountable structures.

H. Ishihara in "The constructive Hahn-Banach theorem, revisited" presents a new proof of an approximate version of the separation theorem by using the Baire category theorem, and a proof of an approximate version of the (1-dimensional) dominated extension theorem in the framework of Bishop constructive mathematics. K. Meer's chapter entitled "Metafinite model theory and real number computations" surveys research done in the area of descriptive complexity theory in relation with the Blum-Shub-Smale model of real number computations.

D. Skordev surveys some technical results in neat computability in his chapter titled "Moschovakis extension of multi-represented spaces". M. I. Soskova's chapter reviews results in "The theory of the enumeration degrees, definability, and automorphisms".

The papers, written by experts in their subjects, are surveys on various topics, some focussed on historical or philosophical topics, others, in fact most, on mathematical results. The technical parts are presented in rigorous manner and are written for an educated reader in their specific topics; they are less inviting to a more generally educated public or students. The relevance to main stream mathematics, one of the objectives of the book, is weak, with two notable exceptions: the articles by Downey and Ishihara.